

9.1

Analysing Graphs of Linear Relations

Focus on...

After this lesson, you will be able to...

- describe patterns on the graph of a linear relation
- create a table of values using the points on a graph

Literacy Link

A *relationship* is a pattern formed by two sets of numbers.



Graphs have been around for a long time. The pictorial writings of Aboriginal peoples and the drawings in the tombs of ancient Egypt are two examples. Graphs have gone through significant changes. Many graphs are now used to show relationships between sets of data.

A table of values also shows a relationship between two quantities. What is an advantage of a graph compared with a table of values?

Explore the Math

How can you represent a linear relation?

Tony's Pizzeria sells medium pizzas for \$9 each. You can order extra toppings for \$1 per topping.

Materials

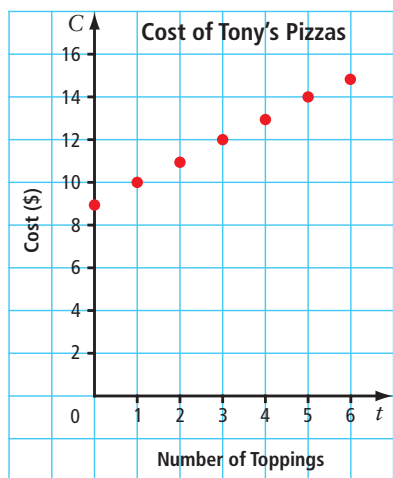
- cardboard circle
- coloured counters

TONY'S PIZZERIA

medium pizza \$9

each extra topping \$1

The graph shows the cost of Tony's medium pizzas.



On the graph, t represents the number of extra toppings and C represents the cost in dollars.

1. a) Use a cardboard circle and coloured counters to model a medium pizza with extra cheese, pineapple, and ham.
 b) Explain how your model could represent the cost of a medium pizza with three toppings. What cost does your model show?
2. Look at the graph. What is the cost of a medium pizza with three toppings? How does this compare with your answer in #1b)?
3. a) From the graph, make a table of values that shows the cost of a medium pizza with zero to six toppings.
 b) What headings did you use for your table? Why?

Strategies
Model It

Literacy Link

A *table of values* is a chart showing two sets of related numbers.

Reflect on Your Findings

4. a) What are three ways you can represent data?
 b) Which way do you prefer? Why?

You can arrange a table of values horizontally or vertically. In a horizontal table, the top row should show the x -coordinates from a graph.

x	1	2	3	4
y	5	10	15	20

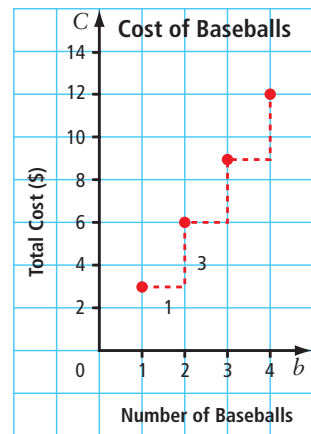
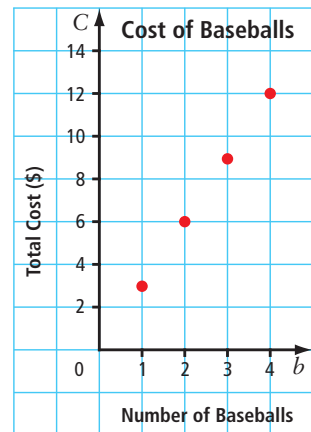
In a vertical table, the first column should show the x -coordinates from a graph.

x	y
1	5
2	10
3	15
4	20

Example 1: Make a Table of Values From a Graph

The graph shows the total cost in relation to the number of baseballs you buy.

- Describe patterns you see on the graph.
- Make a table of values from the graph.
- If the relationship continues, what is the cost of 14 baseballs?



Solution

- The patterns can be described in the following ways:
 - The graph provides data on the cost of baseballs. One ball costs \$3, two balls cost \$6, three balls cost \$9, ...
 - The points appear to lie in a straight line.
 - The graph shows that to move from one point to the next, you go one unit horizontally and three units vertically.

Number of Baseballs, b	1	2	3	4
Total Cost, C (\$)	3	6	9	12

- The graph shows that the cost increases by \$3 for each baseball purchased. Let b represent the number of baseballs. The cost could be represented by $3b$.

$$\begin{aligned} \text{Cost of 14 baseballs} &= 3(14) \\ &= 42 \end{aligned}$$

The cost of 14 baseballs is \$42.

Tech Link

You can use a spreadsheet program to create a table.

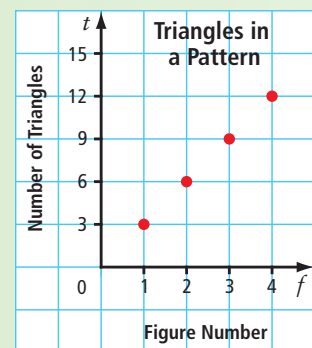
Strategies

Solve an Equation

Show You Know

The graph shows the number of triangles in relation to the figure number in a pattern.

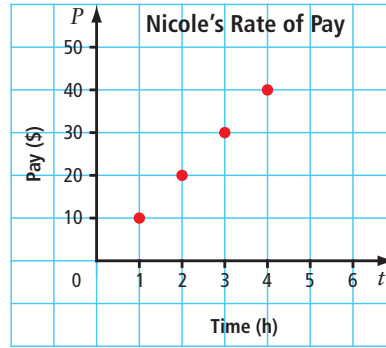
- Describe patterns you see on the graph.
- Draw a triangle pattern that matches the graph.
- Make a table of values from the graph.
- If the pattern continues, how many triangles are in Figure 99?



Example 2: Analyse Data on a Graph of a Linear Relation

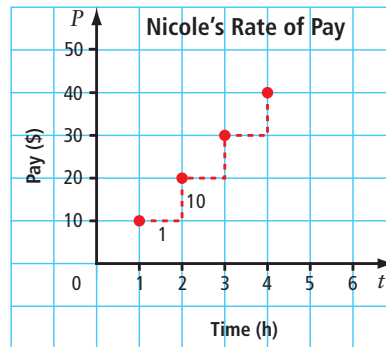
The graph shows Nicole's rate of pay based on the number of hours she has worked at her part-time job.

- Describe patterns you see on the graph.
- What is Nicole's hourly rate of pay? How do you know?
- Make a table of values from the graph.
- Is it possible to have points between the ones on the graph? Explain.



Solution

- The patterns can be described in the following ways:
 - The graph provides data on the pay Nicole receives for each hour worked. The pay for 1 h is \$10, the pay for 2 h is \$20, the pay for 3 h is \$30, ...
 - The points appear to lie in a straight line. The graph shows a linear relation.
 - The graph shows that to move from one point to the next, you go one unit horizontally and ten units vertically.



- Nicole's hourly rate of pay is \$10. The graph shows that Nicole's pay increases by \$10 for each hour that she works.

c)

Time, t (h)	0	1	2	3	4
Pay, P (\$)	0	10	20	30	40

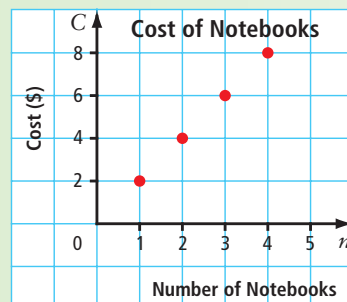
- It may be possible to have points between the ones on the graph. For example, Nicole could get paid for working for $3\frac{1}{2}$ h, and then a point could be shown between (3, 30) and (4, 40) on the graph.

Show You Know

Chad is buying notebooks at Bob's Bargain Store.

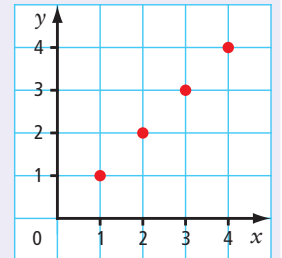
The graph shows the cost of notebooks.

- Describe patterns you see on the graph.
- What is the cost per notebook? How do you know?
- Make a table of values from the graph.
- Is it possible to have points between the ones on the graph? Explain.



Literacy Link

A linear relation is a pattern made by a set of points that lie in a straight line.

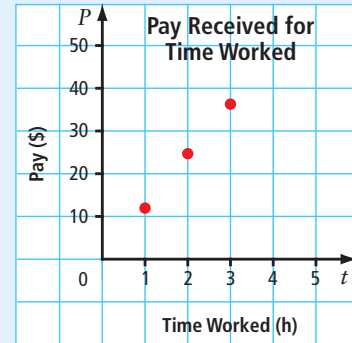


Key Ideas

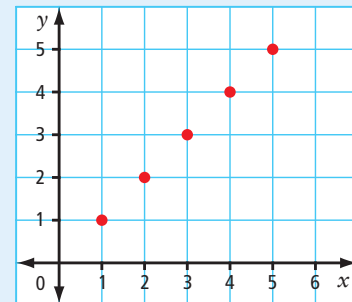
- You can use the coordinates of the points on a graph to make a table of values.
- The top row or left column in a table of values has the same label as the horizontal axis. The second row or right column has the same label as the vertical axis.

Time Worked, t (h)	1	2	3
Pay, P (\$)	12	24	36

Time Worked, t (h)	Pay, P (\$)
1	12
2	24
3	36

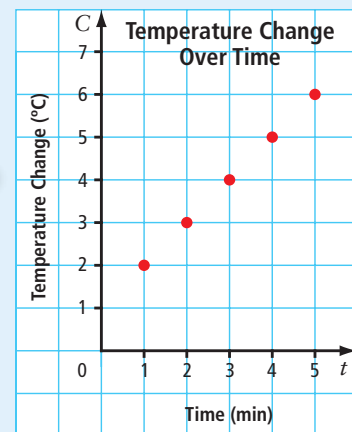


- When you describe a pattern, tell what it relates to, where it starts, and how it changes.
- A linear relation is a pattern made by a set of points that lie in a straight line.



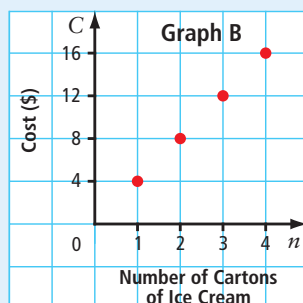
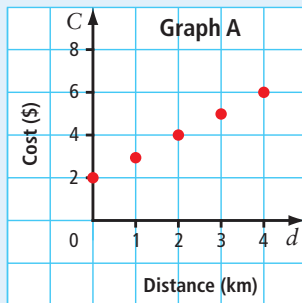
- Sometimes, it is possible to have points between the ones on a graph. Ask yourself, “Does it make sense to have other values between those on the graph?”

Could there be points between the ones on the graph? For example, is it possible to determine the temperature at $2\frac{1}{2}$ min? Is it possible to read a temperature as part of a degree, such as 3.5°C ?



Communicate the Ideas

1. Tell whether you think it is reasonable to have points between the ones on each graph. Explain your answer.



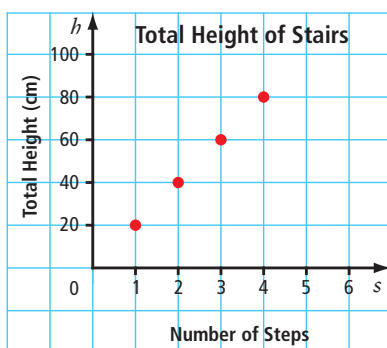
2. Draw a graph of a linear relation. Use integer values only. Label your graph. Write a brief description that matches the information on your graph.
3. Use an example to show one way that a graph and a table of values are different and one way that they are similar.

Check Your Understanding

Practise

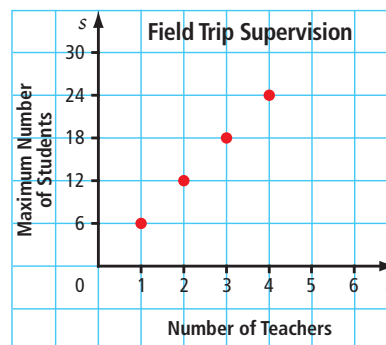
For help with #4 and #5, refer to Example 1 on page 334.

4. The graph shows the increase in total height for each step of a staircase.



- a) Describe patterns you see on the graph.
- b) Make a table of values from the graph.
- c) If the pattern continues, what is the total height on step 10?

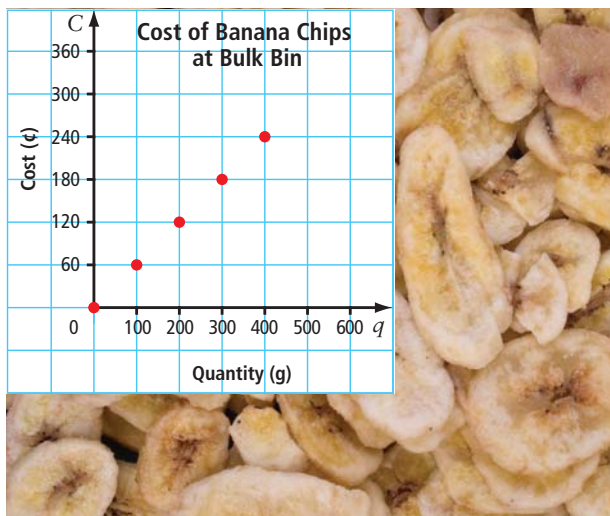
5. The graph shows the maximum number of students allowed on a field trip based on the number of teachers available to supervise.



- a) Describe three patterns on the graph.
- b) Make a table of values from the graph.
- c) If there are eight teachers available for a field trip, what is the maximum number of students who can go?

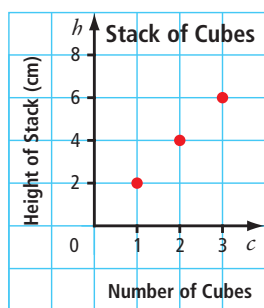
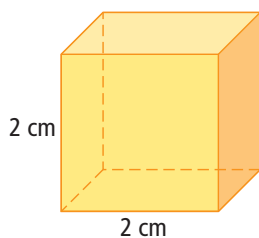
For help with #6 and #7, refer to Example 2 on page 335.

6. Tessa and Vince go shopping at Bulk Bin. The graph shows the cost of banana chips.



- Describe patterns shown on this graph. Does the graph show a linear relation? Explain.
- Make a table of values from the graph.
- Is it reasonable to include a point on the graph that shows the cost of 250 g of banana chips? Explain.

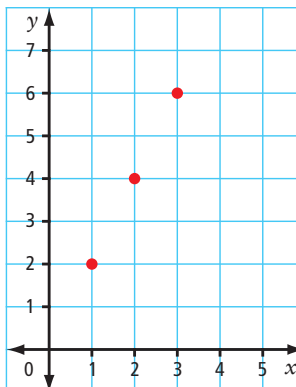
7. The graph shows the height of a stack of cubes in relation to the number of cubes.



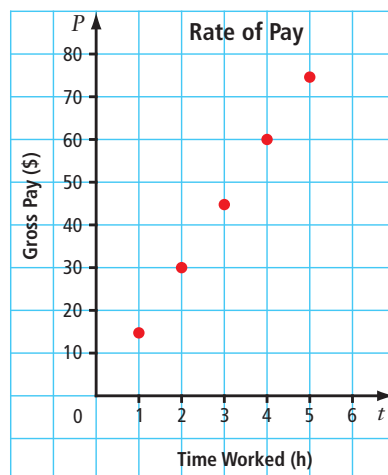
- Describe patterns on the graph. Does the graph show a linear relation? Explain.
- Make a table of values from the graph.
- Is it reasonable to include a point for $c = 2.5$? Explain.

Apply

8. a) Make a table of values for the ordered pairs on the graph.

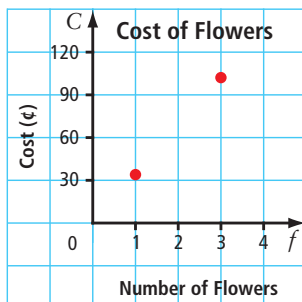


- Assume the pattern continues. Extend your table, using the next three whole number values for x .
 - Describe the patterns on the graph.
 - What is the value of y when the value of x is 9?
9. The graph shows the rate of pay based on the number of hours worked.



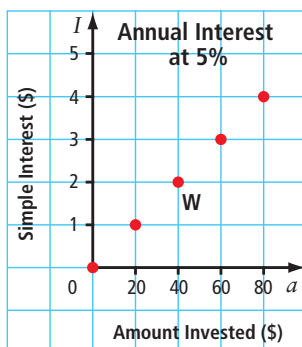
- Make a table of values from the graph.
- What is the hourly rate of pay shown on this graph?
- Do you think it is reasonable to include a point for $t = 3.5$ h?

10. The graph shows part of a linear relation that represents the cost to purchase sugar flowers for a cake.



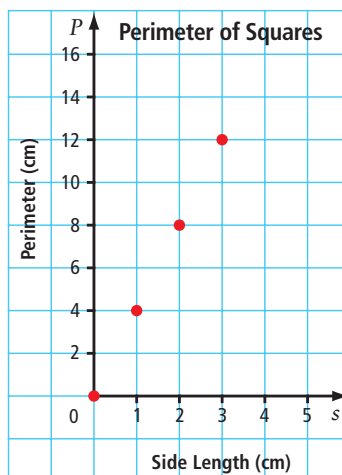
- a) Is it reasonable to have points between the ones on the graph? Explain your answer.
- b) How many points could there be between the two shown on the graph? Explain your answer.

11. The graph shows the simple interest for one year at 5% for different dollar amounts invested.



- a) What are the coordinates for point W?
- b) What does each coordinate for point W represent?
- c) Describe patterns on the graph.
- d) If the pattern continues, what is the simple interest earned on \$180 after one year?

12. The graph shows the perimeter of a square in relation to the length of its side.



- a) Copy the table and fill in the missing values for s and P .

Side Length, s (cm)	Perimeter, P (cm)
0	0
1	4
2	
	12
4	
5	
	32
28	
	124

- b) Describe the patterns on the graph.
- c) Are any other points possible between those shown on the graph? Explain.
- d) Does the graph represent a linear relation? Explain.

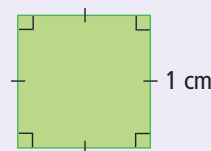
Literacy Link

Perimeter of a Square

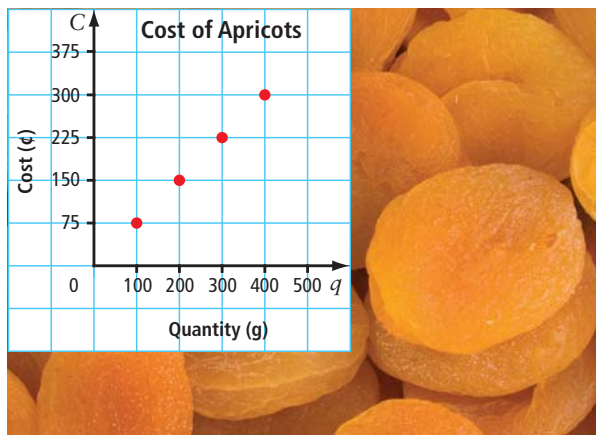
$$P = 1 + 1 + 1 + 1$$

$$P = 4$$

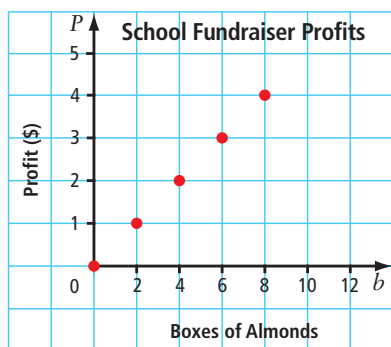
The perimeter is 4 cm.



13. The graph shows the cost of dried apricots at Bulk N Save.

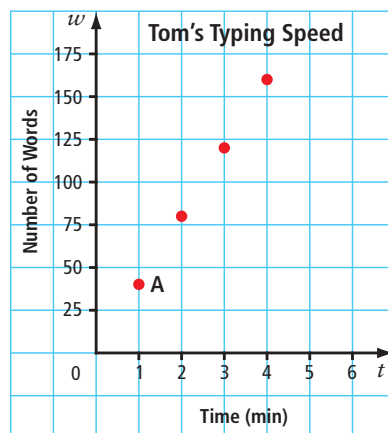


- Make a table of values from the graph.
 - Describe patterns on the graph.
 - Use the graph to estimate the cost of 350 g of dried apricots.
 - What is the actual cost of 350 g of dried apricots? Round your answer to the nearest cent.
 - Compare your answers in parts c) and d).
14. The graph below represents the relationship between the number of boxes of almonds a student sells for a school fundraiser and the profit in dollars for the school.

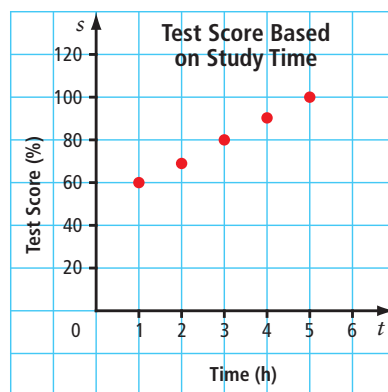


- Make a table of values from the graph.
- Describe three patterns on the graph.
- How much profit does the school make on two boxes of almonds?
- What is the value of P when the value of b is 2? How does this answer relate to your answer in part c)?

15. The following graph represents the number of words Tom can type in relation to the time in minutes.



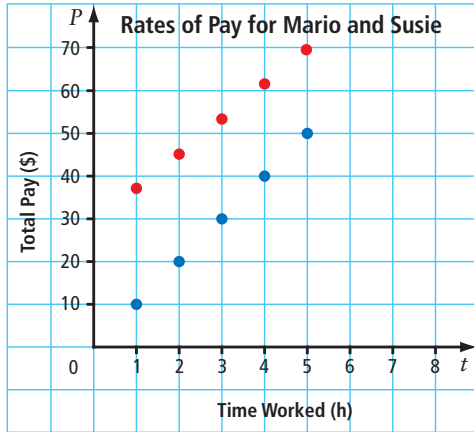
- For the ordered pair $(2, 80)$ tell what each coordinate represents.
 - What is the typing speed in words per minute for point A?
 - Does this graph represent a linear relation? Explain.
 - Would most people's typing speed result in a linear graph? Explain.
16. Alena gathered data comparing the amount of time she spent studying for her tests and the marks she received.



- Make a table of values from this graph.
- Does the graph appear to be a linear relation? Why?
- Will Alena's scores continue increasing at this same rate with more and more time spent studying? Why?

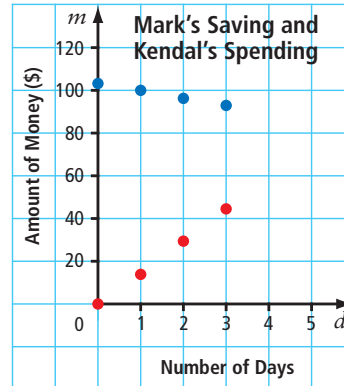
Extend

17. At Ciao Restaurant, Mario works in the kitchen for \$10 an hour. Susie works as a server. She gets \$30 monthly clothing allowance plus \$8 an hour.



- Which colour of points provides information about Susie's wages?
- Make a table of values with three columns showing Time Worked (h) and Total Pay (\$) for Susie and Mario.
- If the pattern continues, at what point will the two sets of points meet?

18. Mark begins saving \$15 a day to buy a three-month gym membership for \$90. Kendal has \$105 in the bank and spends \$5 a day to go to the gym. The graph shows how much money they have over the first three days.



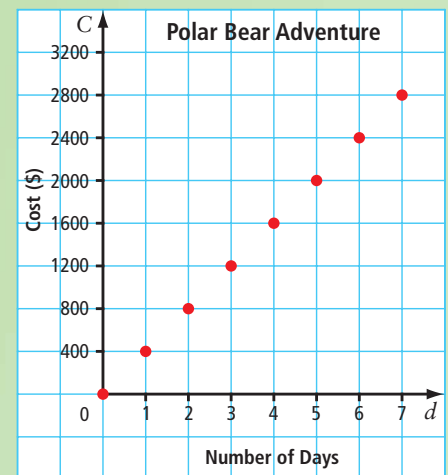
- Which colour of points provides information about Mark?
- When will Kendal run out of money?
- When will Mark have saved enough money to buy the membership?

MATH LINK

Whatever adventure you are looking for, it can be found in Western and Northern Canada. Adventures include polar bear tours in Churchill, Manitoba, aurora borealis adventures in the Northwest Territories, and white-water rafting in Yukon Territory.

You are going on a polar bear adventure tour. The graph shows the cost of the trip.

- Describe any patterns on the graph.
- Make a table of values from the graph.
- Assume the pattern continues. Extend your table of values to include the cost of a tour for eight days.
- Suppose it is possible to upgrade to better accommodations for a one-time fee of \$300 plus the regular \$400 per day. Make a new table of values for deluxe tours that last from one to eight days.
- Compare the data in the two tables of values. How are they similar? How are they different?



9.2

Patterns in a Table of Values

Focus on...

After this lesson, you will be able to...

- identify relationships in a table of values
- decide if a table of values represents a linear relation
- graph points represented by values in a table



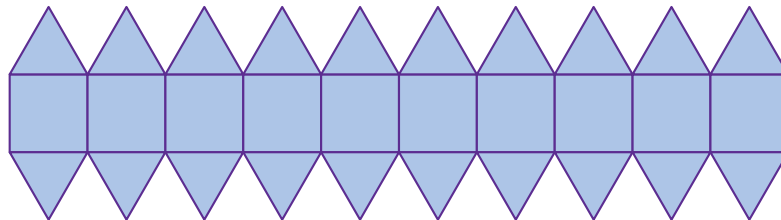
As humans, we see patterns all around us. It is a way for us to make sense of our world. However, different people may see patterns in different places and in different ways. In the picture, Kendra describes a pattern she sees in the wallpaper. What patterns can you identify?

Explore the Math

Materials

- grid paper 

How can you represent patterns?



1. What are three patterns you see in the design?
2. How many squares and triangles are in this design?
3. The design can be lengthened or shortened. Each section of the design must have only complete squares and triangles. If this design has ten sections, what does one section of the design look like?

4. Make a table of values like the one shown. Complete your table for one to four sections in the design.

Number of Sections	Number of Vertical Segments	Number of Horizontal Segments	Number of Slanting Segments
1			
2			
3			
4			

vertical |
horizontal —
slanting / or \

5. a) Use words to describe the relationship between the number of sections and the number of vertical segments.
 b) Choose variables to represent the number of sections and the number of vertical segments. Why did you choose these letters?
 c) Draw a graph to show the relationship between the number of sections and the number of vertical segments. Label the axes with the variables you chose in part b), and with the titles Number of Sections and Number of Vertical Segments.
 d) Describe in words the relationship between the variables.
 e) What is an expression for the number of vertical segments in terms of the number of sections?
6. a) Draw a graph to show the relationship between the number of sections in the design and the data in one of the other columns in the table. Label each axis with a variable and a title.
 b) What is an expression that represents the data in the column you chose in terms of the number of sections in the design?

Literacy Link

A *variable* is a letter that represents an unknown quantity. For example, in $3a - 5$, the variable is a .

It can be helpful to choose variables that are meaningful. For example, t for time and s for score.

Literacy Link

An *expression* is any single number or variable, or a combination of operations (+, −, ×, ÷) involving numbers and variables. An expression does not include an equal sign. The following are examples of expressions:

5 r $8t$
 $x + 9$ $2y - 7$

Reflect on Your Findings

7. a) What ways did you use to represent patterns in this activity?
 b) Which parts of the activity did you find most challenging? Why?

Web Link

To practise your graphing skills, go to www.mathlinks8.ca and follow the links.

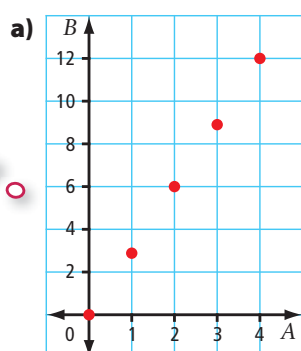
Example 1: Identify the Relationship in a Table of Values

The pattern in the table of values represents a linear relation.

A	B
0	0
1	3
2	6
3	9
4	12

- Graph the ordered pairs in the table of values.
- What is the difference in value for consecutive A -values? for consecutive B -values?
- Describe in words the relationship between the values for A and B .
- What is an expression for B in terms of A ?

Solution



Note that the axes are labelled A and B .

- b) The difference in consecutive A -values is 1. Find the difference in consecutive B -values.

$$3 - 0 = 3 \qquad 9 - 6 = 3$$

$$6 - 3 = 3 \qquad 12 - 9 = 3$$

The difference in consecutive B -values is 3.

Mentally check with values for A in the table.

M E

$$3 \times 1 = 3$$

$$3 \times 2 = 6$$

$$3 \times 3 = 9$$

$$3 \times 4 = 12$$

- When A increases by 1, B increases by 3.
- Use the difference in B -values of 3. Each B -coordinate is 3 times its A -coordinate. An expression for B is $3 \times A$, or $3A$.

The relationship can be written in different ways:

- in words B is 3 times A .
- as an ordered pair $(A, 3A)$
- as an expression $3A$

Show You Know

The table of values represents a linear relation.

x	1	2	3	4	5	6	7
y	5	10	15	20	25	30	35

- Graph the ordered pairs in the table of values.
- What is the difference in value for consecutive x -values? What is the difference in value for consecutive y -values?
- Describe in words the relationship between the x -values and y -values.
- What is an expression for y in terms of x ?

Tech Link

You can use a spreadsheet program to make the graph.

Example 2: Use a Table to Determine a Linear Relation

For each table of values below, answer the following questions:

- What is the pattern in the values for the first variable in each table?
- What is the difference in consecutive values for the second variable in each table? Is the difference within each table the same?
- Graph each set of ordered pairs. Which relations are linear?
- How does your answer in part c) compare with your answer in part b)?

Table 1

x	2	4	6	8
y	3	7	11	15

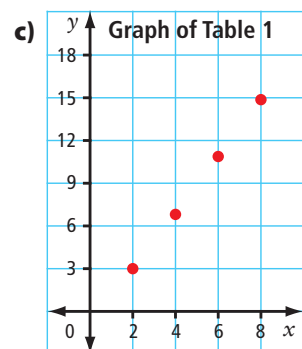
Table 2

m	1	2	3	4
n	1	4	7	8

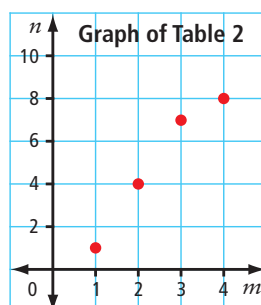
Solution

- a) Table 1: The x -coordinates differ by 2.
 Table 2: The m -coordinates differ by 1.
 The consecutive values of the first variables differ by the same amount within each table.

- b) Table 1: The y -coordinates differ by 4. The difference is the same.
 Table 2: The n -coordinates differ by 3, 3, and 1. The difference is not the same.



The relation is linear.



The relation is not linear.

- d) Table 1: The difference in the y -coordinates is the same and the relation is linear.
 Table 2: The difference in the n -coordinates is not the same and the relation is not linear.

Show You Know

Determine whether each table represents a linear relation. Explain how you know. Check each answer by graphing the ordered pairs.

Table 1

A	1	4	7
B	1	10	19

Table 2

p	0	1	2	3
q	0	1	3	6

Strategies

Look for a Pattern

Literacy Link

If consecutive values for the first variable do not have the same difference, it is difficult to tell from the table whether the relationship is linear. You may be able to tell by drawing a graph.

Example 3: Use a Table of Values in Solving a Problem

Photo World charges \$3 for the first enlargement and \$2 for each additional enlargement.

- Make a table of values showing the cost in relation to the number of enlargements for one to five enlargements.
- Is this a linear relation? Why?
- What is an expression for the cost in relation to the number of enlargements?
- What is the cost of 15 enlargements?



Solution

a) Number of Enlargements, n	1	2	3	4	5
Cost, C (\$)	3	5	7	9	11

- The difference between consecutive numbers of enlargements is 1. Determine the difference for consecutive costs.
 $5 - 3 = 2$
 $7 - 5 = 2$
 $9 - 7 = 2$
 $11 - 9 = 2$
Consecutive costs differ by 2.
Each consecutive number of enlargements differs by the same value.
Each consecutive cost differs by the same value.
The relation is linear.
- The cost increases by \$2 for each additional enlargement. Let n represent the number of enlargements. $2n$ will be part of an expression for the cost. Check using a value for n from the table: $n = 4$.
 $2(4) = 8$
The value of 8 is not the same as the value of 9 in the table. You need to add 1 to get from 8 to 9.
An expression for the cost is $2n + 1$, where n is the number of enlargements.
- Substitute $n = 15$ into the expression $2n + 1$.
 $2(15) + 1 = 31$
The cost of 15 enlargements is \$31.

Mentally check $2n + 1$ with other values for n in the table.

M E

$$2(1) + 1 = 3$$
$$2(2) + 1 = 5$$
$$2(3) + 1 = 7$$
$$2(5) + 1 = 11$$

Show You Know

Sky sells magazine subscriptions. She receives \$20 for every five subscriptions she sells. The table shows the relationship between the number of subscriptions she sells and the pay she receives.

Number of Subscriptions, n	0	5	10	15	20	25
Pay, P (\$)	0	20	40	60		

- Copy and complete the table.
- Is this a linear relation? How can you tell?
- What is an expression for Sky's pay in relation to the number of subscriptions she sells?
- How much does Sky receive for selling 40 subscriptions?

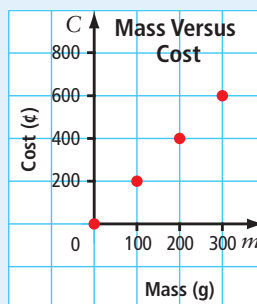
Key Ideas

- The following are some ways that you can represent a linear relation.

- Table of values:

Mass, m (g)	0	100	200	300
Cost, C (¢)	0	200	400	600

- Graph:



- Words: The cost in cents is 2 times the mass in grams.
- Ordered pair: $(m, 2m)$
- Expression: The cost in cents is $2m$, where m is the mass in grams.

- You can sometimes tell from a table whether the relationship is linear.

Table 1

p	2	3	4	5
q	7	13	19	25

Table 2

p	q
20	31
40	27
60	23
80	19

You can tell that the relationships in the above tables are linear because both of the following statements are true:

- Each consecutive value for p changes by the same amount.
- Each consecutive value for q changes by the same amount.

Communicate the Ideas

1. Giselle and Tim are discussing the table of values shown.

<i>m</i>	3	5	7	9
<i>a</i>	1	3	5	7

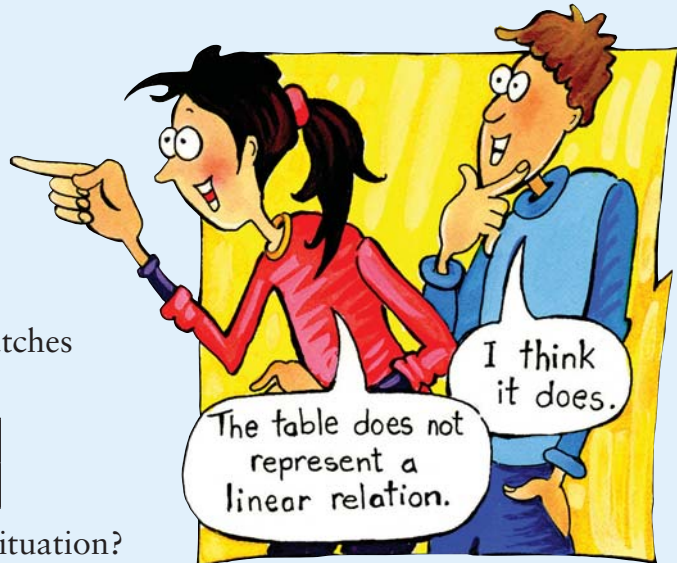
Who is correct? How do you know?

2. a) Describe a real-life situation that matches the pattern in the table of values.

<i>n</i>	1	2	3	4
<i>p</i>	1	3	5	7

- b) What do n and p represent in your situation?

3. You are given a table of values. Can you sometimes, always, or never tell whether the relationship is linear without drawing the graph? Use examples to support your answer.



Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 344.

4. Graph the ordered pairs in the table of values.

<i>a</i>	<i>d</i>
1	5
2	8
3	11
4	14

5. Draw a graph using the ordered pairs in the table of values.

<i>w</i>	<i>t</i>
1	1
4	7
7	13
10	19

6. The table of values represents a linear relation.

<i>x</i>	0	1	2	3	4	5
<i>a</i>	0	4	8	12	16	20

- a) Graph the ordered pairs.
 b) What is the difference in value for consecutive x -values? What is the difference in value for consecutive a -values?
 c) In words, describe the relationship between x and a .
 d) What is an expression for a in terms of x ?

7. The table of values shows a linear relation.

<i>n</i>	3	4	5	6	7
<i>d</i>	18	24	30	36	42

- Graph the ordered pairs.
- What is the difference in value for consecutive n -values? What is the difference in value for consecutive d -values?
- In words, describe the relationship between n and d .
- What is an expression for d in terms of n ?

For help with #8 and #9, refer to Example 2 on page 345.

8. For each table of values, tell whether the relationship is linear. Explain your answer. Check by graphing the ordered pairs.

a)	<i>c</i>	<i>d</i>	b)	<i>x</i>	<i>y</i>
	2	7		0	-3
	3	10		1	1
	4	13		3	9
	5	16		5	17

9. Tell whether the relationship for each table of values is linear. Explain how you know.

a)	<i>p</i>	4	7	10	16
	<i>q</i>	11	17	23	29
b)	<i>x</i>	2	3	4	5
	<i>y</i>	3	2	1	0

For help with #10 and #11, refer to Example 3 on page 346.

10. Mara reads at a rate of 90 words per minute.
- Make a table of values that shows the total number of words Mara reads in one to six minutes. Use whole minutes.
 - Is this a linear relation? Explain.

- What is an expression for the number of words Mara reads in terms of time?
- How many words does Mara read in 12 min?

11. The dosage of a certain medication to be given to a child is related to the child's mass. A 10-kg child receives 50 mg of the medication. For each increase in mass of 1 kg, the child receives an additional 10 mg of the medication.

- Make a table showing the dosage for children with an increase of 1 kg to 10 kg over the mass of 10 kg. Use whole kilograms only.
- Is this a linear relation? Explain how you know.
- What is an expression for the dosage in terms of the increase in mass over 10 kg?
- What is the dosage for a child with a mass of 27 kg?
- Can your table of values start at 0 kg? Why or why not?

Apply

12. Evan has \$6 in quarters and dimes.



- Name five combinations of quarters and dimes that Evan might have.
- Make a table of values showing the relationship between quarters and dimes. Include five pairs of values in your table.
- Draw a graph. Is the relationship between quarters and dimes linear in this example? Explain.
- What is the largest possible number of dimes? of quarters?

13. Divers experience an increase in pressure as they dive deeper below sea level. The table shows the relationship between depth in metres and pressure in atmospheres.

Depth (m)	Pressure (atm)
0	1
10	2
20	3
	4
	5
	6

- a) Copy and complete the table.
 b) Draw a graph for the ordered pairs in the table. How should your axes be labelled?
 c) Divers sometimes become dizzy when the pressure exceeds 5 atm. Below what depth do divers tend to become dizzy?

14. The following pattern continues.

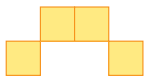


Figure 1

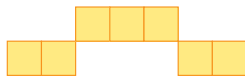


Figure 2

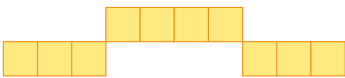
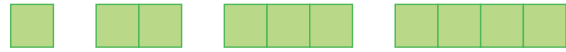


Figure 3

- a) Make a table of values showing the figure number and the number of squares for the first six figures.
 b) Write an expression showing the number of squares in terms of the figure number. What does your variable represent?
 c) How many squares would appear in Figure 20?
 d) How many more squares are in Figure 20 than in Figure 10? Show two ways to find the answer.

15. The following pattern of squares continues.



- a) Copy and complete the table of values below that shows the relationship between the number of squares and the perimeter of each figure.

Number of Squares	1	2		
Perimeter (cm)	4	6		

- b) Draw a graph from the table of values.
 c) Describe the patterns on the graph.
 d) What is an expression for the perimeter in terms of the number of squares?
 e) If the pattern continues, what is the perimeter when there are 50 squares?
16. As you climb a mountain, the temperature drops 1°C for every 150 m of increased height.

- a) Copy and complete the table to show the relationship between height and temperature if the temperature at the bottom of the mountain is 20°C .

Height (m)	0	150	300	450	600	750
Temperature ($^\circ\text{C}$)	20					

- b) Graph the ordered pairs
 c) Is the relationship linear?
 d) How high have you climbed if the temperature is 13°C ?
17. A skydiver jumps from an airplane. The table provides data for the period of time shortly after the parachute opens, relating time in seconds to total distance descended in metres.

Time (s)	Distance (m)
10	300
11	354
12	408
13	462



- a) Predict how a graph for these ordered pairs would look.
- b) Graph the ordered pairs. Was your prediction correct?
- c) Describe the graph in words.

18. A community centre has a new banquet hall. The centre charges \$5 per person to rent the hall.

- a) Make a table of values showing the rental cost for 20, 40, 60, 80, and 100 people.
- b) Graph the ordered pairs.
- c) What is an expression for the rental cost in terms of the number of people?

Extend

19. The community centre in #18 changes the cost for renting its banquet hall. The centre now charges \$50 plus \$5 per person.

- a) Make a table of values showing the rental cost for 20, 40, 60, 80, and 100 people.
- b) Graph the ordered pairs. How does the graph differ from the graph in #18?

- c) What is an expression for the rental cost in terms of the number or letter of people? What does each number or letter in your expression represent?

20. Jamal is renting snowboard equipment.



a) Copy and complete the table.

Number of Additional Days	0	1	2	3	4	5
Rental Cost (\$)						

- b) What is an expression for the rental cost in terms of the number of additional days?
- c) What is the cost to rent the snowboard for a total of ten days? What might be a better option for Jamal instead of renting for ten days?

MATH LINK

Have you ever gone for a canoe trip on the waters of one of Canada's national parks? You are planning a canoe trip. The cost to rent a canoe is \$40 a day. A national park pass for one week costs \$36.

- a) Make a table of values showing the total cost for the pass and canoe for a trip from one to seven days.
- b) Graph the ordered pairs in your table of values. Is this a linear relation? Explain.
- c) What is an expression for the cost based on the number of days?
- d) Think of a linear relationship to do with canoeing. Show the relationship using a table of values, a graph, words, and an expression.



9.3

Linear Relationships

Focus on...

After this lesson, you will be able to...


- graph from a formula
- graph from an equation using integers
- solve problems using the graph of a linear relation



Scientists look for patterns. They collect experimental data in charts and tables. Then, they graph the data. These graphs are pictures that show mathematical relationships in the data. When scientists write a formula for the relationship, it may represent a new scientific discovery! What are some formulas that you are familiar with?

Explore the Math

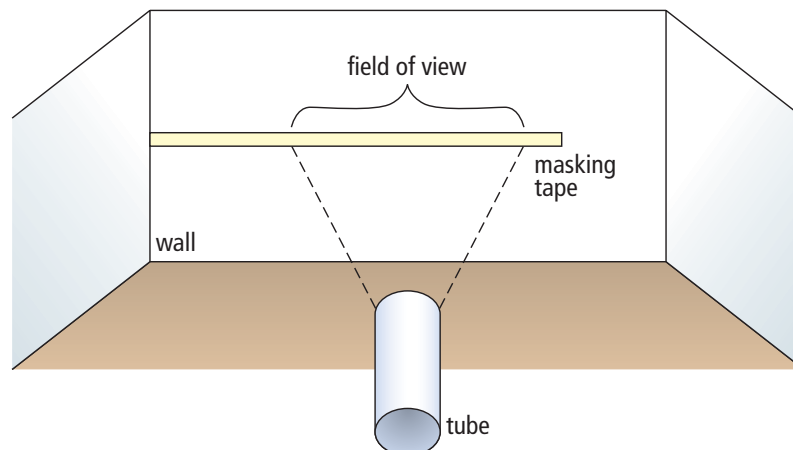
Materials

- metre stick or measuring tape
- masking tape
- different-sized tubes such as from paper towel rolls
- grid paper 

How do you determine a relationship?

Work in a group.

1. Beginning in a corner, place about 2 m of masking tape horizontally on a wall at eye level. Start at the corner and mark off the masking tape in 5-cm intervals.



2. On the floor, measure and mark five distances from the wall. Start at 100 cm and increase the distance by 30 cm each time.
3. Copy the table into your notebook.

Distance From Wall (cm)	Width of Field of View (cm)

4. Predict how the width of your field of view through a tube will change as your distance from the wall increases.
5. For each distance from the wall, measure the width that you can see through the tube. Record the data in the table.
6. Write the results of your experiment as a set of ordered pairs. Graph the ordered pairs.
7. Compare your results with your prediction.
8. Repeat steps 3 to 7, using a different-sized tube.

Reflect on Your Findings

9. a) Do your data show a linear relation? Explain.
b) Why might the points on your graph line up as they do?

Example 1: Graph From a Linear Formula

Sound travels 1500 m/s in salt water. This relationship can be expressed by the formula $d = 1500t$, where d is the distance travelled, in metres, and t is the time, in seconds.

- a) Make a table of values. Use integers only.
- b) Graph the ordered pairs in your table of values.
- c) Is it reasonable to have points between the ones on this graph? Explain.
- d) Calculate the distance sound travels in 3.5 s.

Literacy Link

A *formula* is a mathematical statement that represents the relationship between specific quantities. An example is $C = \pi \times d$, where C is the circumference and d is the diameter of the circle.

Solution

- a) Choose the numbers to use in the table of values. For example, $t = 0, 1, 2, 3, 4, 5, 6$

Time, t (s)	0	1	2	3	4	5	6
Distance, d (m)	0	1500	3000	4500	6000	7500	9000

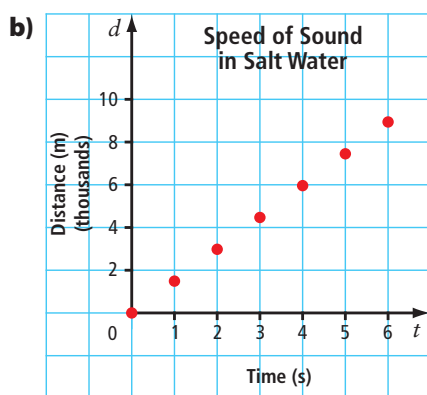
To determine d , substitute each t value into the formula. For example, substitute $t = 3$.

$$d = 1500t$$

$$d = 1500(3)$$

$$d = 4500$$

Choose at least four values for t in the table of values. Check that they are reasonable. It is not possible for t to be a negative integer since time cannot be negative. Check that $t = 0$ is reasonable in the context. When you are graphing, it is often useful to include zero as a value on the horizontal axis.



- c) It is reasonable to have points between the ones on the graph. For example, it is possible to determine the distance at 2.5 s, and it is possible to determine the time at 4800 m.
- d) Substitute 3.5 for t in the formula $d = 1500t$.
- $$d = 1500(3.5)$$
- $$d = 5250$$
- The distance travelled at 3.5 s is 5250 m.
The estimate of 5000 m is close to the calculated distance of 5250 m.

When you rent a tool or machine, you are charged by the hour. For example, if you rent a backhoe and use it for $6\frac{1}{2}$ h, you are charged for 7 h.

Show You Know

Paula rents a lawnmower for \$8 per hour. A formula representing this relationship is $C = 8t$, where C is the rental cost, in dollars, and t is the time, in hours.

- Make a table of values.
- Graph the ordered pairs in your table of values.
- Is it reasonable to have points between those on the graph? Explain.
- Calculate the cost to rent for 12 h.

Example 2: Graph From a Linear Equation Using Integers

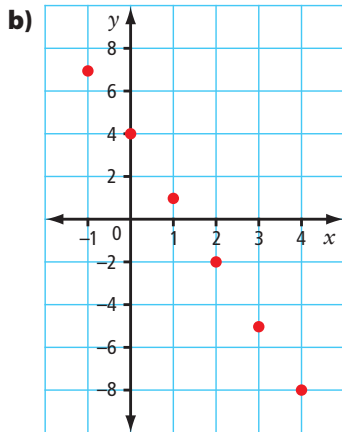
Use the linear equation $y = -3x + 4$ to complete the following steps.

- Make a table of values. Use positive and negative integers and zero for values of x .
- Graph the ordered pairs in your table of values.
- Determine the value for y in the ordered pair $(11, y)$.

Solution

- Choose the numbers to use in the table of values. For example,
 $x = -1, 0, 1, 2, 3, 4$

x	-1	0	1	2	3	4
y	7	4	1	-2	-5	-8



Since no context is given, many values for x and y are possible. Choose values for x that are the same distance apart. For example, you could choose $x = -1, 0, 1, 2, 3, 4$ or $x = -2, 0, 2, 4, 6, 8$.

- Substitute $x = 11$ into the equation $y = -3x + 4$.
 $y = -3x + 4$
 $y = -3(11) + 4$
 $y = -33 + 4$
 $y = -29$

The value of the y -coordinate is -29 .
The ordered pair is $(11, -29)$.

Show You Know

Use $y = 2x + 3$ to answer the following.

- Make a table of values. Use integers only.
- Graph the ordered pairs in your table of values.
- What are the coordinates for the point that would lie on the y -axis?

Literacy Link

An *equation* is a mathematical statement with two expressions that have the same value. The two expressions are separated by an equal sign. For example,
 $x + 2 = 3$
 $y - 7 = -4$
 $3a - 2 = a + 2$
 $b = 4$

Strategies

Solve an Equation

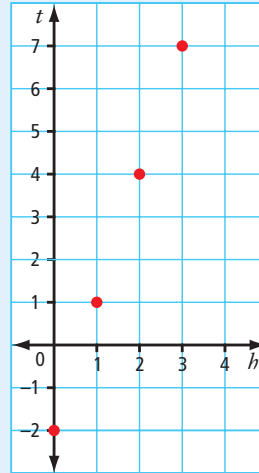
What is the value for x of a point that lies on the y -axis?

Key Ideas

- You can graph a linear relation represented by a formula or an equation.
 - First, make a table of values. Check that the values in the table are reasonable.
 - Then, graph using the ordered pairs in the table.

$$t = 3h - 2$$

h	t
0	-2
1	1
2	4
3	7



- Whenever possible, choose variables that are meaningful. For example, h for height and t for temperature.

Communicate the Ideas

- What do you notice about the coordinates for points on the x -axis?
 - What do you notice about the coordinates for points on the y -axis?
- The equation $y = 2x - 1$ represents a linear relation. A table of values for $y = 2x - 1$ has been started below. Show two different ways to find the value of the missing y -coordinate.

x	0	1	2	3	4
y	-1	1	3	5	

- When you choose values for a table of values, what considerations might influence your choice?
 - Why is it often useful to use zero as one of your x -values?
- Give an example of a linear relation found in real life. Then, give an example of a linear relation without a context.
 - Make a table of values for each linear relation. Use integers only.
 - Graph each set of ordered pairs.
 - How are these graphs similar? How are they different?

Check Your Understanding

Practise

For help with #5 and #6, refer to Example 1 on pages 353–354.

5. A long-distance telephone plan can be represented as $C = 6t$, where C is the cost in cents and t is the time in minutes.

1	2	3	Only 6¢ per minute anytime for calls across Canada!
4	5	6	
7	8	9	
*	0	#	
			CallCanada

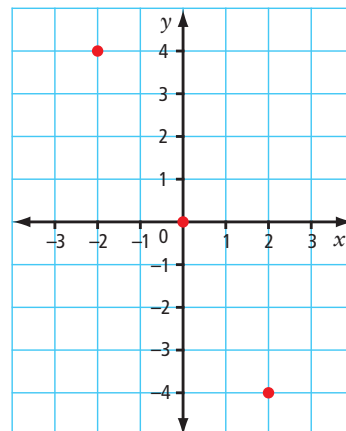
- a) Make a table of values, using at least four whole number values for t .
- b) Graph the ordered pairs.
- c) If part minutes are rounded up to the next whole minute, is it reasonable to have points between the ones on your graph? Explain.
6. An animal shelter hires you to walk dogs for \$5 per dog. The relationship between the money you make and the number of dogs you walk can be expressed as $W = 5d$, where W is the wage you make, in dollars, and d is the number of dogs you walk.



- a) Make a table of values.
- b) Graph the ordered pairs.
- c) Is it reasonable to have points between the ones on your graph? Explain.

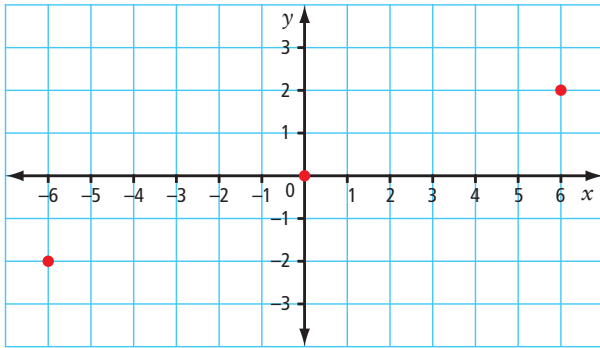
For help with #7 to #12, refer to Example 2 on page 355.

7. Evaluate each equation using the given value.
- $y = 5x - 3$ when $x = 6$
 - $y = x - 8$ when $x = 5$
 - $y = -5x$ when $x = -2$
 - $y = x$ when $y = 25$
8. Evaluate $y = 7x + 3$ using each value.
- $x = 1$
 - $x = -4$
 - $x = 0$
 - $y = 17$
9. Make a table of values for each equation using $x = -2, 0, 2, 4$. Draw each graph.
- $y = 3x + 2$
 - $y = x - 5$
 - $y = -4x$
 - $y = 7 - x$
10. Make a table of values for each equation. Choose at least four integer values for x . Graph each set of ordered pairs.
- $y = 2x$
 - $y = 3x - 1$
 - $y = -4x + 5$
 - $y = 5 - x$
11. The graph below represents part of the linear relation $y = -2x$.



- Use the equation to calculate the y -coordinate when $x = -1$.
- What is the value of y in $(-4, y)$?

12. The graph represents part of the linear relation $y = \frac{x}{3}$.



- What are the coordinates for the point that lies on the y -axis?
- Use the equation to calculate the y -coordinate when $x = -3$.
- For the point $(-9, y)$, what is the value of y ?

Apply

13. a) Graph the ordered pairs in the table.

x	y
-2	0
1	2
4	4
7	6

- Is it reasonable to assume there are points between the ones on your graph if you have no other information? Why?
14. You are given part of the table of values for a linear relation.
- | x | -3 | -2 | -1 | 0 | 1 | 2 |
|-----|----|----|----|---|---|----|
| y | | | | 6 | 8 | 10 |
- How could you use a pattern to find the missing y -coordinates?
 - Identify the missing y -coordinates.

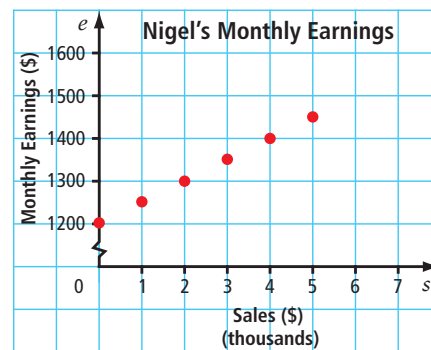
15. In a bulk food store, rice crackers sell for 80¢ per 100 g.
- Copy and complete the table of values.

Mass of Purchase (g)	Cost (¢)
0	
100	
200	
300	

- If you continue the table, what is the next most logical value to use for Mass of Purchase? Explain, using your knowledge of linear relations and patterns.
- Graph the ordered pairs.



16. The graph shows Nigel's monthly earnings.



- If Nigel does not make any sales, what are his monthly earnings?
- Nigel has sales of \$4000 in one month. How much does he earn?
- Nigel earns \$1500 in a month. What are his sales?

17. You can buy work gloves from The Fix-It Store's web site according to the formula $C = 5g + 2$, where C is the cost in dollars and g is the number of pairs of gloves.

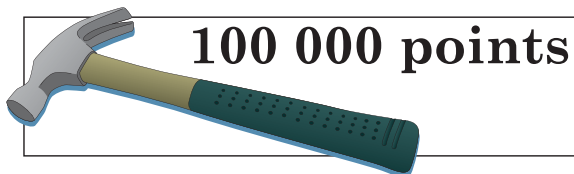
- Make a table to show the number of pairs of gloves purchased in relation to the total cost. Use five values for g .
- Graph the ordered pairs.
- Does the relation appear linear? Why?
- Are there other points possible between the ones on the graph? Explain.
- What might 2 represent in the formula?

18. George is a carpenter. He has a rewards card for the hardware store he uses. He receives 40 points for every \$1 he spends. He can use the points to get savings on future purchases.

- Copy and complete the table showing the dollar amount spent in relation to the number of points received.

Amount Spent (\$)	Points Received
1	40
2	
3	
4	
5	

- How many points does George receive for spending \$100?
- George can get a hammer for 100 000 points. How much money does he have to spend to get the hammer?



19. Shandi determined the mass of five pieces of a type of metal. The table shows her results. The relationship is linear. Shandi made one error in finding the masses.

Volume (cm³)	8	9	10	11	12
Mass (g)	88	99	110	121	144

- Use patterns to show how to find the incorrect mass. What is the correct mass?
- Draw points on a graph using Shandi's results in the table above.
- How could you use your graph to show which value is incorrect?

Extend

20. A taxi company in Edmonton charges \$3 for the first 210 m plus 20¢ for each additional 210 m.

- What is the cost for a trip of length 2100 m? of length 4.41 km?
- Make a table of values showing the relationship between the distance in metres and the cost in cents. Use integers only.
- Graph the ordered pairs.
- Is it a linear relation? Why?

21. Describe the pattern in each table of values. Then, graph each set of ordered pairs.

- | | | | | | |
|----------|---|---|---|---|---|
| t | 0 | 1 | 2 | 3 | 4 |
| d | 2 | 3 | 4 | 5 | 6 |

- | | | | | |
|----------|---|---|---|----|
| x | 8 | 7 | 6 | 2 |
| y | 4 | 3 | 2 | -2 |

22. a) Complete the table using the relationship "multiply x by 2 and then add 3 to get y ."

x	-2	-1	0	1	2	3	11
y							

- Is it a linear relation? Explain.