8.1 Exploring Integer Multiplication

Focus on...

After this lesson, you will be able to...

- ❑ multiply integers using integer chips

Did You Know?

The Columbia Icefield is a major source of fresh water. Melt water from the icefield feeds rivers that flow to the Arctic Ocean, the Pacific Ocean, and Hudson's Bay.

The Columbia Icefield is the largest mass of ice in North America below the Arctic Circle. The icefield lies across the Alberta–British Columbia border in the Rockies. Six large glaciers flow from the icefield. One of them, the Athabasca Glacier, is a popular tourist destination in Jasper National Park.

The Athabasca Glacier has been melting for over a century. The front edge or "snout" of the glacier has been receding at an average of approximately 12 m per year. At that rate, how far would it recede in four years?

Explore the Math

Materials

- red and blue integer chips

Literacy Link

Representing Integers

Integer chips are coloured disks that represent integers. In this book, one red chip represents +1, and one blue chip represents −1.

+1 −1

How can you use integer chips to multiply two integers?

1. Multiplication can be expressed as a repeated addition.

$(+3) \times (+2) = (+2) + (+2) + (+2)$

a) Use red integer chips to model the addition $(+2) + (+2) + (+2)$.

b) Copy and complete the multiplication statement $(+3) \times (+2) =$ ■.

2. a) Express $(+4) \times (+3)$ as a repeated addition.

b) Use red integer chips to model the addition.

c) Copy and complete the multiplication statement $(+4) \times (+3) =$ ■.

3. a) Express $(+3) \times (-5)$ as a repeated addition.

b) Use blue integer chips to model the addition.

c) Copy and complete the multiplication statement $(+3) \times (-5) =$ ■.

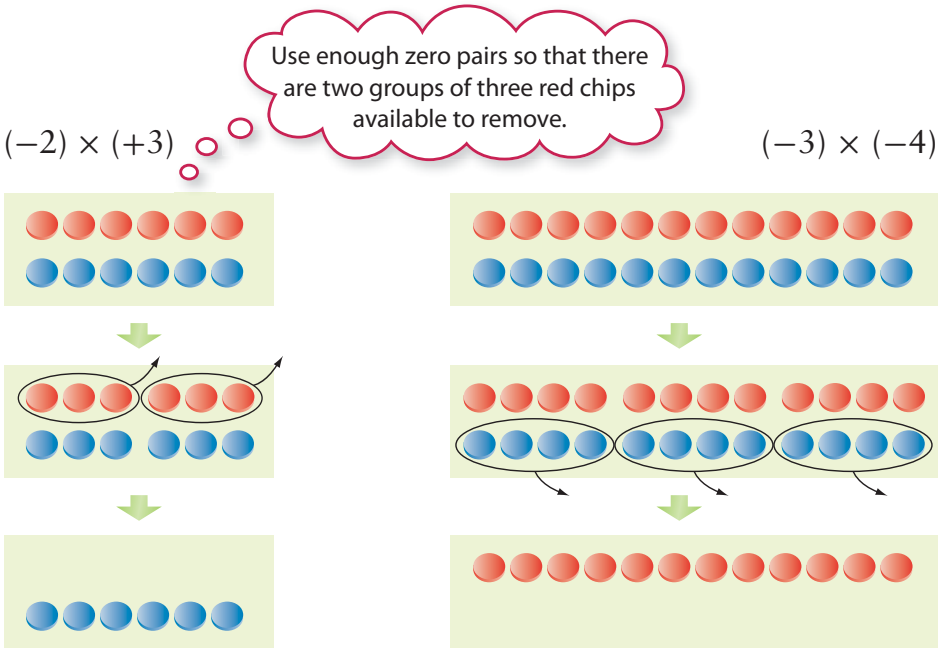
4. Use integer chips to model each multiplication. Copy and complete each multiplication statement.

a) $(+3) \times (+5) =$ ■ **b)** $(+2) \times (+3) =$ ■

c) $(+4) \times (-3) =$ ■ **d)** $(+3) \times (-4) =$ ■

5. Can you use the same method as in #1 to #4 to complete the multiplication $(-2) \times (+3)$ or the multiplication $(-3) \times (-4)$? Explain.

6. a) The diagrams show how to model the multiplications from #5 by using zero pairs. Describe each model.



b) Copy and complete each multiplication statement.

$(-2) \times (+3) =$ ■

$(-3) \times (-4) =$ ■

7. Use integer chips to model each multiplication. Copy and complete each multiplication statement.

a) $(-4) \times (-3) =$ ■ **b)** $(-5) \times (+2) =$ ■

c) $(-2) \times (+4) =$ ■ **d)** $(-1) \times (-4) =$ ■

Reflect on Your Findings

8. How can you use integer chips to multiply two integers? In your description, state when you use zero pairs. How do you determine the number of zero pairs to use?

Literacy Link

Understanding Multiplication

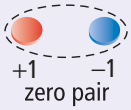
The product of 4 and 2 is 8, because $4 \times 2 = 8$.

The multiplication statement $4 \times 2 = 8$ means that 4 groups of 2 make 8. You can also think of 4×2 as the repeated addition $2 + 2 + 2 + 2$.

Literacy Link

Modelling With Zero Pairs

A zero pair is a pair of integer chips, with one chip representing $+1$ and one chip representing -1.



A zero pair represents zero because $(+1) + (-1) = 0$.

Any whole number of zero pairs represents zero.

Example 1: Multiply Using Integer Chips

Determine each product using integer chips. Copy and complete each multiplication statement.

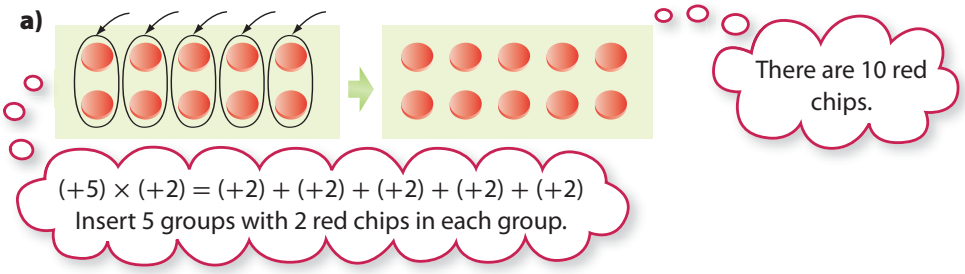
a) $(+5) \times (+2)$ **b)** $(+6) \times (-2)$ **c)** $(-3) \times (+2)$ **d)** $(-2) \times (-4)$

Solution

Strategies
Model It

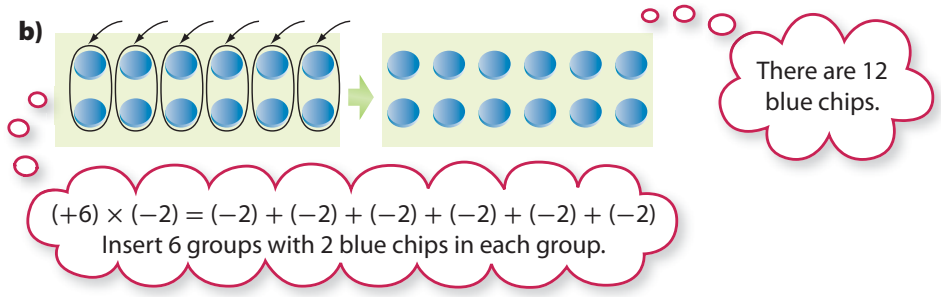
Strategies
What other strategy could you use?

a)



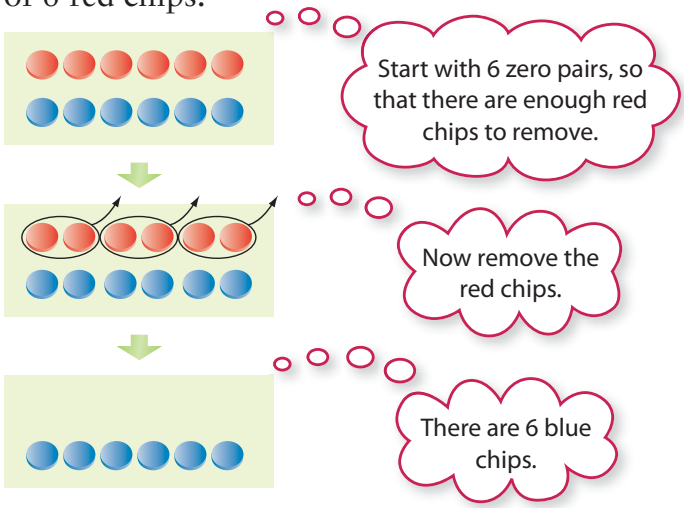
The product is $+10$.
The multiplication statement is $(+5) \times (+2) = +10$.

b)



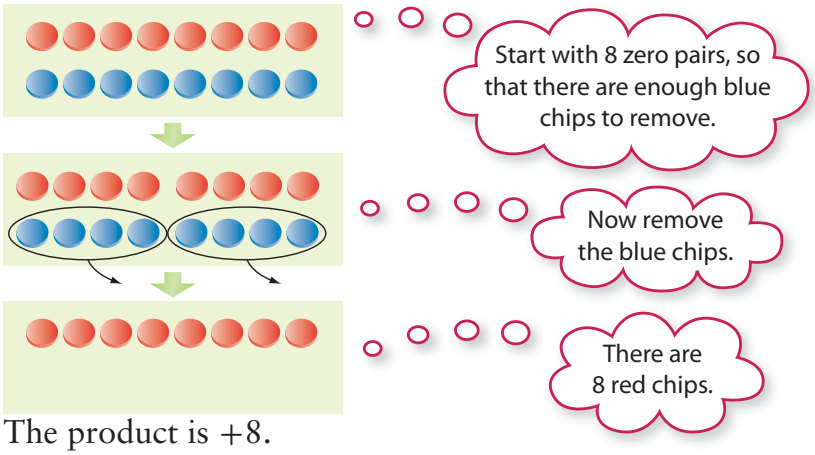
The product is -12.
The multiplication statement is $(+6) \times (-2) = -12$.

c) The negative sign in -3 shows that you need to remove 3 groups. Each group will contain 2 red chips. So, you need to remove a total of 6 red chips.



The product is -6.
The multiplication statement is $(-3) \times (+2) = -6$.

d) The negative sign in -2 shows that you need to remove 2 groups. Each group will contain 4 blue chips. So, you need to remove a total of 8 blue chips.



The product is $+8$.
The multiplication statement is $(-2) \times (-4) = +8$.

Show You Know

Determine each product using integer chips. Use diagrams to show your thinking.

a) $(+4) \times (+2)$ **b)** $(+5) \times (-2)$ **c)** $(-4) \times (+2)$ **d)** $(-6) \times (-1)$

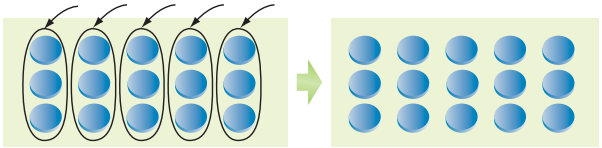
Example 2: Apply Integer Multiplication

For 5 h, the temperature in Flin Flon fell by 3 °C each hour. What was the total change in temperature?

Solution

Use the multiplication of two integers to represent the situation.
Represent the time of 5 h by the integer $+5$.
Represent the 3 °C decrease in each hour by the integer -3.
The total change in temperature can be represented by the expression $(+5) \times (-3)$.

Multiply $(+5) \times (-3)$ using integer chips.



The product is -15.
The total change in temperature was a decrease of 15 °C.

Show You Know

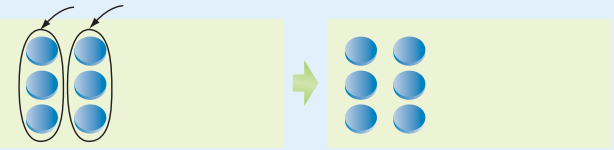
For 4 h, the temperature in Victoria fell by 2 °C each hour. What was the total change in temperature?

Did You Know?

Flin Flon lies along the Manitoba–Saskatchewan border. The town is named after a fictional character called Professor Josiah Flintabbatey Flonatin. He was the hero of a science fiction novel called *The Sunless City*. In this novel, Josiah explored a bottomless lake in a submarine and discovered a tunnel lined with gold. Flin Flon was named after him because of the large mineral deposits discovered there.

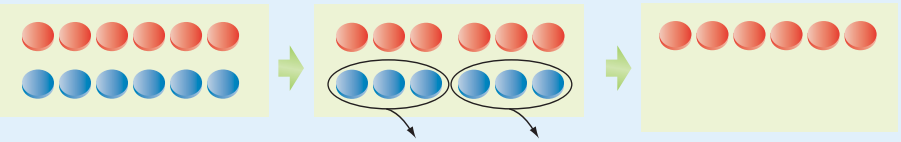
Key Ideas

- To model the multiplication of an integer by a positive integer, you can insert integer chips of the appropriate colour.



$(+2) \times (-3) = -6$

- To model the multiplication of an integer by a negative integer, you can remove integer chips of the appropriate colour from zero pairs.



$(-2) \times (-3) = +6$

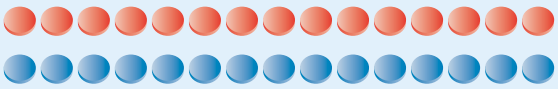
Communicate the Ideas

1. David said that he could model the multiplication $(+3) \times (-7)$ using 3 red chips and 7 blue chips.

a) Do you agree with David? Explain.

b) What chips would you use to model $(+3) \times (-7)$? Explain.

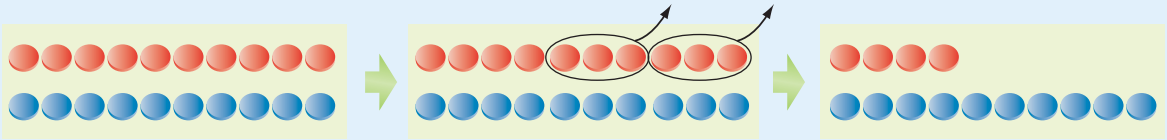
2. To model $(-3) \times (-5)$, Raini places 15 zero pairs on her desk.



a) Why did she use 15 zero pairs?

b) What should she do next?

3. a) Paolo models $(-2) \times (+3)$ as shown in the diagram. He determines the correct product, -6. Explain why his method works.



b) Could Paolo model the product if he started with 4 zero pairs? Explain.

4. Jasmine said that she did not need any integer chips to multiply $0 \times (+2)$ or to multiply $(-3) \times 0$. Explain her thinking.

Check Your Understanding

Practise

For help with #5 to #14, refer to Example 1 on pages 288–289.

5. Write each repeated addition as a multiplication.

a) $(+1) + (+1) + (+1) + (+1) + (+1)$

b) $(-6) + (-6)$

6. Write each expression as a multiplication.

a) $(+7) + (+7) + (+7)$

b) $(-4) + (-4) + (-4) + (-4)$

7. Write each multiplication as a repeated addition.

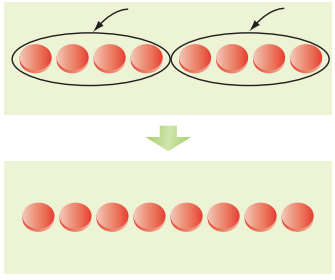
a) $(+3) \times (+8)$ **b)** $(+5) \times (-6)$

8. Write each expression as a repeated addition.

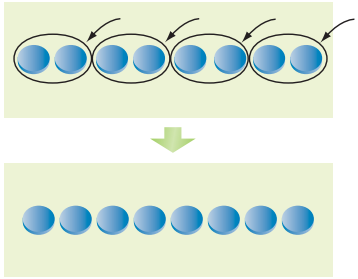
a) $(+7) \times (+2)$ **b)** $(+4) \times (-9)$

9. What multiplication statement does each set of diagrams represent?

a)

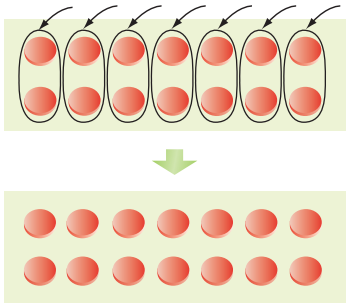


b)

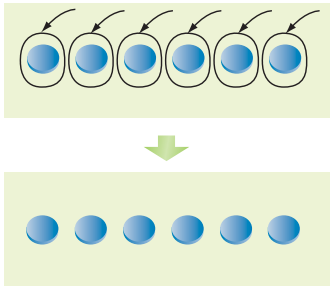


10. What multiplication statement does each set of diagrams represent?

a)

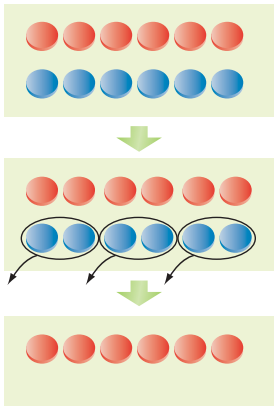


b)

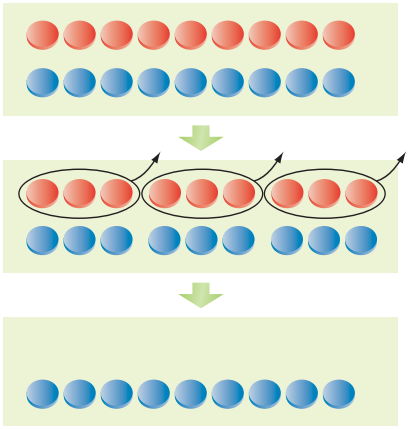


11. What multiplication statement does each set of diagrams represent?

a)

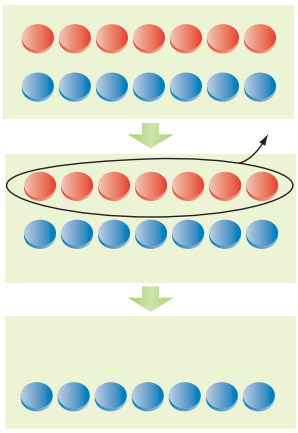


b)

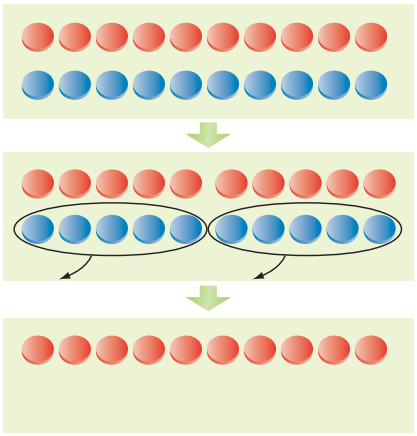


12. What multiplication statement does each set of diagrams represent?

a)



b)



13. Copy and complete each multiplication statement.

a) $(+4) \times (+6)$ **b)** $(+7) \times (-2)$

c) $(-1) \times (+5)$ **d)** $(-8) \times (-2)$

Apply

For help with #14 to #17, refer to Example 2 on page 289.

14. Use the multiplication of two integers to represent each situation. Then determine the product and explain its meaning.

a) The temperature increased for 6 h at 2 °C per hour.

b) Ayesha repaid some money she owed in 4 instalments of $8 each.

15. An aircraft descends at 3 m/s for 12 s. Use the multiplication of two integers to represent the situation. How far does the aircraft descend?

16. A building has 10 storeys above ground and 3 storeys below ground. Each storey has a height of 4 m.

a) What is the total height of the building above ground?

b) What is the total depth of the building below ground?

17. An oil rig is drilling a well at 2 m/min. How deep is the well after the first 8 min?

18. Does doubling an integer always result in an integer of greater value? Explain.

Extend

19. In a magic square, the numbers in each row, column, and diagonal have the same sum. This is called the magic sum.

a) What is the magic sum for this magic square?

+2	+3	−2
−3	+1	+5
+4	−1	0

b) Multiply each integer in the square by -2. Is the result another magic square? If so, what is the new magic sum?

c) Create a magic square with a magic sum of -12.

20. Arrange the following numbers of $+1$s and -1s in the small squares on a three-by-three grid so that each row, column, and diagonal has a product of -1.

a) six $+1$s, three -1s

b) four $+1$s, five -1s

8.2 Multiplying Integers

Focus on...

After this lesson, you will be able to...

- ❑ determine integer products using a number line
- ❑ apply a sign rule when multiplying integers

Birds called sooty shearwaters have the longest known migration of any animal. Huge flocks of these birds leave their breeding grounds in New Zealand as winter approaches. They fly across and around the Pacific Ocean to take advantage of summer in the Northern Hemisphere. Some of them visit the coastal waters of British Columbia. The birds head south again as winter approaches in the North. Scientists have measured the birds' annual migration at about 70 000 km.

Sooty shearwaters feed by diving into the ocean to catch fish, squid, and krill. The birds dive to an average depth of 14 m. Their deepest dives are about five times as deep as that. How could you use integers to determine the depth of their deepest dives?

Did You Know?

For many years, scientists thought that the arctic tern was the distance champion of bird migration. This bird breeds in the Arctic and migrates to Antarctica and back each year. The distance that it covers is at least 35 000 km.

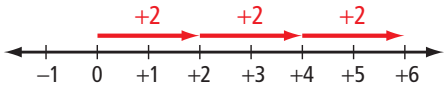
Explore the Math

Materials

- red and blue integer chips

How can you multiply two integers?

1. The diagram shows how you can model the multiplication $(+3) \times (+2)$ using a number line.



a) How are the two integers in the multiplication $(+3) \times (+2)$ shown in the diagram?

b) Model $(+3) \times (+2)$ using integer chips. What is the product?

c) How does the number line show the product?

2. a) Model the multiplication $(+4) \times (-3)$ using a number line. Explain your reasoning.

b) What is the product? Explain how you know.

3. Can you use the same method as in #1 or #2 to model $(-3) \times (+2)$ or $(-4) \times (-3)$ using a number line? Explain.

4. a) Copy the table. Use a suitable model to help you complete each multiplication statement.

$(+6) \times (+2) =$ ■	$(+2) \times (+6) =$ ■
$(+4) \times (-5) =$ ■	$(-5) \times (+4) =$ ■
$(-4) \times (-3) =$ ■	$(-3) \times (-4) =$ ■

b) Compare the two multiplication statements on each row of the completed table. What can you conclude about the order in which you can multiply two integers? Test your conclusion on some other integer multiplications.

c) From your answer to part b), describe a way to determine $(-3) \times (+2)$ using a number line.

5. a) Copy each of the following statements. Use your results from the table in #4 to complete each statement using the word "positive" or the word "negative."

The product of two integers with the same sign is ■.
The product of two integers with different signs is ■.

b) Test your statements from part a) on some other integer multiplications.

Reflect on Your Findings

6. a) How can you use a number line to multiply two integers? In your description, state any limitations of your method.

b) How can you use the signs of two integers to help determine their product?

Example 1: Multiply Integers

Calculate.

a) $(+3) \times (+4)$ **b)** $(+2) \times (-9)$ **c)** $(-5) \times (+6)$ **d)** $(-6) \times (-4)$

sign rule

(for multiplication)

- the product of two integers with the same sign is positive
- the product of two integers with different signs is negative

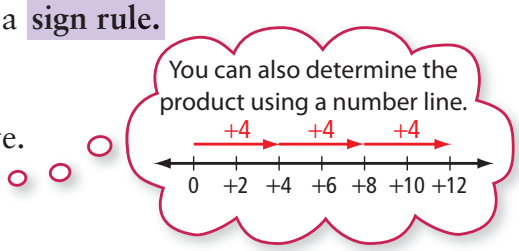
Solution

Multiply the numerals and then apply a **sign rule.**

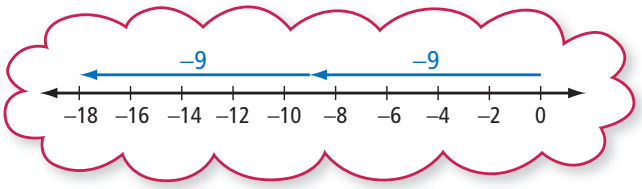
a) $3 \times 4 = 12$

The integers $+3$ and $+4$ have the same sign, so the product is positive.

$(+3) \times (+4) = +12$



b) $2 \times 9 = 18$
The integers $+2$ and -9 have different signs, so the product is negative.
$(+2) \times (-9) = -18$



c) $5 \times 6 = 30$
The integers -5 and $+6$ have different signs, so the product is negative.
$(-5) \times (+6) = -30$

d) $6 \times 4 = 24$
The integers -6 and -4 have the same sign, so the product is positive.
$(-6) \times (-4) = +24$

Show You Know

Calculate.

a) $(+4) \times (+7)$ **b)** $(+3) \times (-10)$ **c)** $(-8) \times (-2)$ **d)** $(-4) \times (+9)$

Example 2: Apply Integer Multiplication

Tina supports her favourite charity with an automatic deduction of $35/month from her bank account. Estimate and then calculate the total of her deductions in a year?

Solution

Use the multiplication of two integers to represent the situation.
Represent the $35 deduction each month by the integer -35. Represent the number of monthly deductions in a year by the integer $+12$.

You could also write $(-35) \times (+12)$, because the order of multiplication does not matter.

The total of the deductions can be represented by the product $(+12) \times (-35)$.

Estimate the product.
$12 \times 35 \approx 10 \times 40$
so $(+12) \times (-35) \approx (+10) \times (-40)$
≈ -400

The integers $+12$ and -35 have different signs, so the product is negative.

Multiply $(+12) \times (-35)$ using the sign rules.
$(+12) \times (-35) = -420$
The total of her deductions in a year is $420.

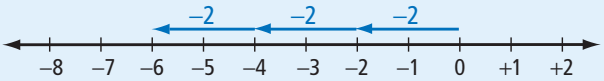
The negative sign shows that Tina's account has decreased by $420.

Show You Know

Duane instructs his bank to deduct $65 per month from his bank account and transfer the money into an investment account. What is the total of his deductions in 18 months?

Key Ideas

- You can model the multiplication of a positive integer by an integer on a number line.



$(+3) \times (-2) = -6$

- You can multiply two integers by multiplying the numerals and applying the sign rules:
  - The product of two integers with the same sign is positive.
    $(+2) \times (+5) = +10$
    $(-2) \times (-5) = +10$
  - The product of two integers with different signs is negative.
    $(+2) \times (-5) = -10$
    $(-2) \times (+5) = -10$
- Multiplying two integers in either order gives the same result.
  $(-5) \times (+3) = -15$
  $(+3) \times (-5) = -15$

Communicate the Ideas

1. Darcy modelled $(+7) \times (+3)$ on a number line by drawing seven arrows. Ishnan modelled $(+7) \times (+3)$ on a number line by drawing only three arrows. Explain Ishnan's thinking.

2. Justin said, "When I multiply $+5$ by a negative integer, the product is less than $+5$. If I multiply -5 by a negative integer, I think the product should be less than -5." Do you agree with him? Explain.

3. Without doing any calculations, Wei said that -19 and $+27$ have the same product as $+19$ and -27. How did she know?

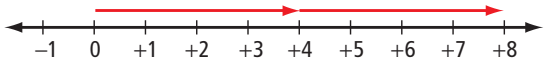
Check Your Understanding

Practise

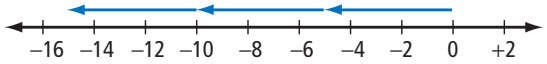
For help with #4 to #9, refer to Example 1 on pages 294–295.

4. What multiplication statement does each diagram represent?

a)

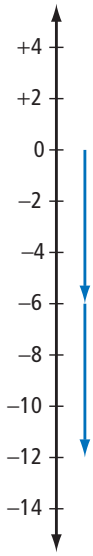


b)



5. What multiplication statement does each diagram represent?

a)



b)

6. Determine each product using a number line.

a) $(+5) \times (+5)$ **b)** $(+3) \times (-6)$

7. Determine each product.

a) $(+4) \times (-7)$ **b)** $(+2) \times (+9)$

8. Determine each product using the sign rules.

a) $(+10) \times (+4)$ **b)** $(+6) \times (-5)$

c) $(-7) \times (+5)$ **d)** $(-8) \times (-4)$

9. Determine each product.

a) $(-6) \times (-6)$ **b)** $(+9) \times (+6)$

c) $(-12) \times (+2)$ **d)** $(+11) \times 0$

For help with #10 to #15, refer to Example 2 on page 295.

10. Estimate and then calculate.

a) $(+17) \times (-24)$

b) $(+37) \times (+22)$

c) $(-72) \times (+15)$

d) $(-28) \times (-47)$

11. Estimate and then calculate.

a) $(-18) \times (-14)$

b) $(-51) \times (+26)$

c) $(+99) \times (+12)$

d) $(+55) \times (+55)$

Apply

For #12 to #15, use the multiplication of two integers to represent each situation.

12. A telephone company offers its customers a $15 discount per month if they also sign up for Internet service. How much is the annual discount?

13. A hot-air balloon is descending at 60 m/min. How far does it descend in 25 min?

14. Ana owns 75 shares of the Leafy Greens Company. One week, the value of each share dropped by 60¢. The next week, the value of each share grew by 85¢. What was the total change in the value of Ana's shares

a) in the first week?

b) in the second week?

c) over the two-week period?

15. To prepare for the weightlessness of space, astronauts train using steep dives on an aircraft. In one dive, the aircraft can descend at 120 m/s for 20 s. How far does the aircraft descend?

16. In the following list of integers, identify the two integers that have the greatest product.

$$+21, -18, +12, +14, -23, -15, +19, -13$$

17. Without evaluating the products, identify the least product. Explain your reasoning.

$(+99) \times (+82)$
$(-99) \times (-82)$
$(+99) \times (-82)$

18. Suppose a friend knows how to multiply positive integers but has never multiplied negative integers.

a) How could you use the following pattern to show your friend how to calculate $(+5) \times (-3)$?

$(+5) \times (+3) = +15$
$(+5) \times (+2) = +10$
$(+5) \times (+1) = +5$
$(+5) \times 0 = 0$
$(+5) \times (-1) =$ ■
$(+5) \times (-2) =$ ■
$(+5) \times (-3) =$ ■

b) Make up a pattern to show your friend how to calculate $(+6) \times (-2)$.

19. a) Can $+4$ be written as the product of two equal integers? Explain.

b) Can -4 be written as the product of two equal integers? Explain.

20. Copy and complete each multiplication statement.

a) $(+6) \times$ ■ $= +18$

b) ■ $\times (-2) = -10$

c) ■ $\times (+3) = -12$

d) $(-4) \times$ ■ $= +16$

21. Complete each statement in as many ways as possible using integers.

a) ■ $\times$ ■ $= +10$

b) ■ $\times$ ■ $= -16$

c) ■ $\times$ ■ $= -24$

22. The sum of two integers is -5. The product of the same two integers is -36. What are the two integers?

23. Write a word problem that you can solve using the expression $(+5) \times (-6)$.

24. Create your own word problem that involves integer multiplication. Make sure that you can solve your problem. Give your problem to a classmate to solve.

Extend

25. Describe each pattern. Then write the next three terms in each pattern.

a) $+1, +3, +9, +27, \ldots$

b) $-1, +2, -4, +8, \ldots$

c) $-2, -4, -8, -16, \ldots$

d) $+2, -8, +32, -128, \ldots$

26. For each statement, describe a situation in which the statement is true.

a) The product of two integers equals one of the integers.

b) The product of two integers equals the opposite of one of the integers.

c) The product of two integers is less than both integers.

d) The product of two integers is greater than both integers.

Literacy Link

Two integers with the same numeral but different signs are called opposite integers. Examples are +5 and −5.

27. a) Identify three consecutive integers whose sum and product both equal zero.

b) Repeat part a) for five consecutive integers.

c) Can you repeat part a) for two consecutive integers or for four consecutive integers? Explain.

28. In a magic multiplication square, the numbers in each row, column, and diagonal have the same product. This is called the magic product.

a) What is the magic product of this square?

+12	−1	+18
−9	−6	−4
+2	−36	+3

b) Multiply each number in the square from part a) by −2. Is the result a magic multiplication square? If so, what is the magic product?

c) Add −5 to each number in the square from part a). Is the result a magic multiplication square? If so, what is the magic product?

29. Write a sign rule for the product of each of the following.

a) an even number of positive integers

b) an odd number of positive integers

c) an even number of negative integers

d) an odd number of negative integers

MATH LINK

The temperature of still, dry air decreases by about 6 °C for each kilometre increase in altitude. A weather balloon was launched from The Pas, Manitoba, on a still, dry day.

a) If the temperature on the ground was +4 °C, what was the approximate temperature 11 km above the ground?

b) If the balloon then descended to 5 km above ground, about how much did the temperature change during the descent?

8.3 Exploring Integer Division

Focus on...

After this lesson, you will be able to...

- ❑ **divide integers using integer chips**

Grizzly bears lose much of their body mass during their winter sleep. A large male bear may enter his den at 300 kg in November. He may lose 75 kg by the time he emerges five months later. How would you represent a loss of 75 kg with an integer? What operation would you use to find the average loss of mass in one month?

Explore the Math

Materials

- red and blue integer chips

Literacy Link

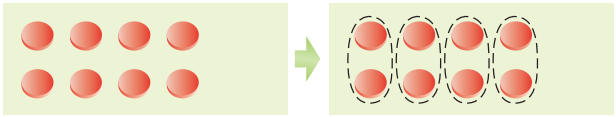
Understanding Division

In the division statement $6 \div 2 = 3$, the dividend is 6, the divisor is 2, and the quotient is 3.

This division statement means that in 6 there are 3 groups of 2. It also means that when 6 is separated into 2 equal groups, there are 3 in each group.

How can you use integer chips to divide two integers?

1. The diagram shows a way to model the division $(+8) \div (+2)$ using red integer chips.



a) Explain how the diagram shows the quotient $(+8) \div (+2)$.

b) Copy and complete the division statement $(+8) \div (+2) = \blacksquare$.

c) Explain how the same diagram can also model $(+8) \div (+4)$.

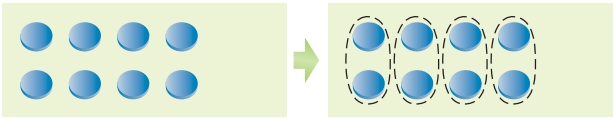
d) Copy and complete the division statement $(+8) \div (+4) = \blacksquare$.

2. a) Use red integer chips to model the division $(+15) \div (+5)$.

b) Copy and complete the division statement $(+15) \div (+5) = \blacksquare$.

c) Write the other division statement that the model can represent.

3. The diagram shows a way to model the division $(-8) \div (-2)$ using blue integer chips.



a) Explain how the diagram shows the quotient $(-8) \div (-2)$.

b) Copy and complete the division statement $(-8) \div (-2) = \blacksquare$.

c) Explain how the same diagram can also model $(-8) \div (+4)$.

d) Copy and complete the division statement $(-8) \div (+4) = \blacksquare$.

4. a) Use blue integer chips to model the division $(-15) \div (-5)$.

b) Copy and complete the division statement $(-15) \div (-5) = \blacksquare$.

c) Write the other division statement that the model can represent.

5. a) Model the division $(-8) \div (+2)$ using integer chips. Explain your method.

b) Copy and complete the division statement $(-8) \div (+2) = \blacksquare$.

Reflect on Your Findings

6. How can you use integer chips to divide two integers?

Example 1: Divide Using Integer Chips

Determine each quotient using integer chips. Copy and complete each division statement.

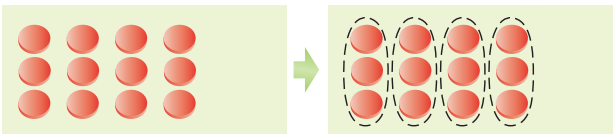
a) $(+12) \div (+3)$

b) $(-12) \div (-3)$

c) $(-12) \div (+4)$

Solution

a)



There are 4 groups, so the quotient is $+4$.
The division statement is $(+12) \div (+3) = +4$.

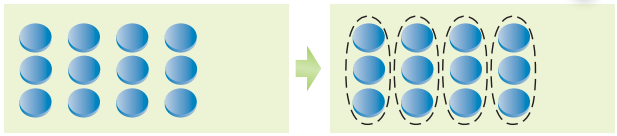
Separate the 12 red chips into groups of 3 red chips and count the number of groups.

Strategies
Model It

Another way to model this division is to separate the 12 red chips into 3 equal groups. There are 4 red chips in each group, so each group represents $+4$.

Strategies
What other strategy could you use?

b)

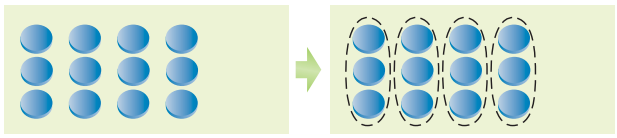


There are 4 groups, so the quotient is +4.
The division statement is $(-12) \div (-3) = +4$.

Separate the 12 blue chips into groups of 3 blue chips and count the number of groups.

You cannot model this division by separating the 12 blue chips into −3 groups.

c)



There are 3 blue chips in each group, so the quotient is −3.
The division statement is $(-12) \div (+4) = -3$.

Separate the 12 blue chips into 4 equal groups and count the number of blue chips in each group.

You cannot model this division by separating the 12 blue chips into groups that each represent +4.

Show You Know

Determine each quotient using integer chips. Use diagrams to show your thinking.

a) $(+14) \div (+7)$ **b)** $(-9) \div (-3)$ **c)** $(-16) \div (+2)$

Example 2: Apply Integer Division

One night, the temperature in Wetaskiwin, Alberta, was falling by 2 °C each hour. How many hours did it take for the temperature to fall 10 °C altogether? Show how you found your answer using integer chips.

Solution

Use the division of two integers to represent the situation.
Represent the 2 °C decrease each hour by the integer −2.
Represent the total decrease of 10 °C by the integer −10.
The number of hours taken can be represented by the expression $(-10) \div (-2)$.
Divide $(-10) \div (-2)$ using integer chips.

The total change divided by the change per hour gives the number of hours.

Separate the 10 blue chips into groups of 2 blue chips. Count the number of groups.

There are 5 groups, so the quotient is +5.
It took 5 h for the temperature to fall 10 °C altogether.

WWW Web Link

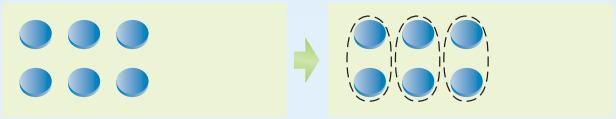
The name *Wetaskiwin* comes from the Cree term *wi-ta-ski-oo cha-ka-tin-ow*, which means "place of peace" or "hill of peace." To find out more about Aboriginal sources of Canadian place names, go to www.mathlinks8.ca and follow the links.

Show You Know

The temperature in Buffalo Narrows, Saskatchewan, was falling by 3 °C each hour. How many hours did it take for the temperature to fall 12 °C altogether? Show how you found your answer using integer chips.

Key Ideas

- You can use integer chips to model integer division.



$(-6) \div (-2) = +3$

$(-6) \div (+3) = -2$

Communicate the Ideas

1. a) Allison modelled the division $(+12) \div (+6)$ by separating 12 red chips into groups of 6. Tyler modelled the same division by separating 12 red chips into 6 equal groups. Explain how they each determined the correct quotient.

b) Explain how each of their methods also models the division $(+12) \div (+2)$.

c) Using blue chips, could you use Tyler's method to model $(-12) \div (+6)$? Explain.

d) Using blue chips, could you use Allison's method to model $(-12) \div (+6)$? Explain.

2. a) Wing modelled the division $0 \div (+4)$ by separating 8 zero pairs into 4 groups. There were 2 zero pairs in each group. Explain how his model shows the quotient.

b) Could you model the same division with a different number of zero pairs? Explain.

c) Would you use integer chips to divide 0 by a positive or negative integer? Explain.

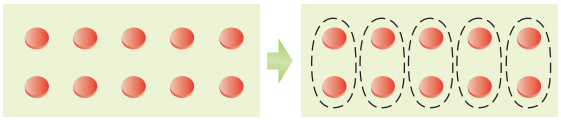
Check Your Understanding

Practise

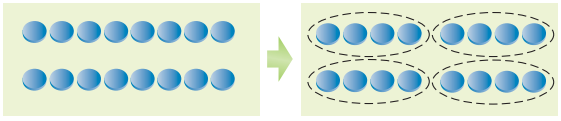
For help with #3 to #8, refer to Example 1 on pages 301–302.

3. Copy each division statement. Use the diagrams to complete it.

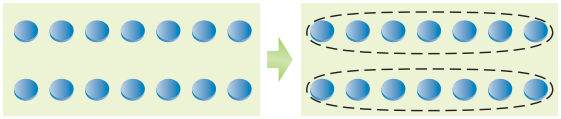
a) $(+10) \div (+2) =$ ■



b) $(-16) \div (-4) =$ ■

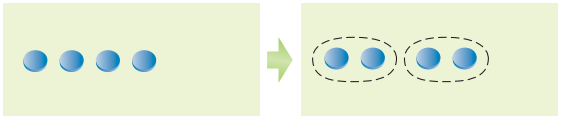


c) $(-14) \div (+2) =$ ■

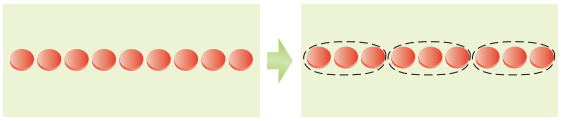


4. Copy each division statement. Use the diagrams to complete it.

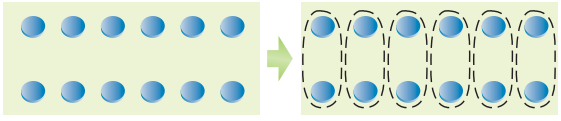
a) $(-4) \div (-2) =$ ■



b) $(+9) \div (+3) =$ ■

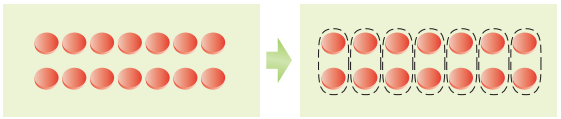


c) $(-12) \div (+6) =$ ■

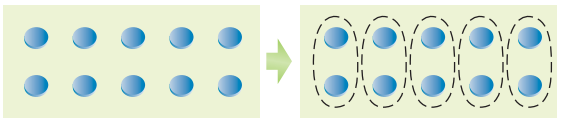


5. Copy both division statements. Use the diagrams to complete them.

a) $(+14) \div (+2) =$ ■
$(+14) \div (+7) =$ ■

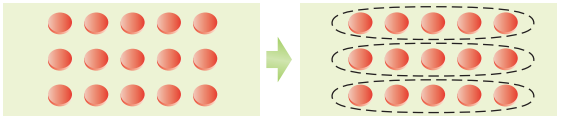


b) $(-10) \div (-2) =$ ■
$(-10) \div (+5) =$ ■

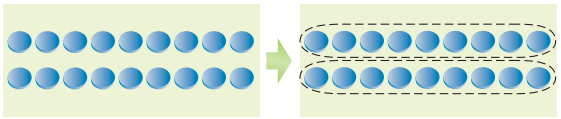


6. Copy both division statements. Use the diagrams to complete them.

a) $(+15) \div (+5) =$ ■
$(+15) \div (+3) =$ ■



b) $(-18) \div (-9) =$ ■
$(-18) \div (+2) =$ ■



7. Determine each quotient using integer chips. Have a partner check your chips. Then copy and complete the division statement

a) $(+16) \div (+4) =$ ■

b) $(-7) \div (+7) =$ ■

c) $(-12) \div (-6) =$ ■

8. Divide using integer chips. Then copy and complete the division statement.

a) $(-20) \div (-10) =$ ■

b) $(-10) \div (+2) =$ ■

c) $(+4) \div (+2) =$ ■

Apply

For help with #9 to #11, refer to Example 2 on page 302. Use the division of two integers to represent each situation and solve the problem.

9. A submarine was diving at 3 m/min. How long did it take to dive 21 m?

10. From 11:00 p.m. to 5:00 a.m., the temperature in Saskatoon fell from −1 °C to −19 °C.

a) What was the change in temperature?

b) What was the change in temperature per hour? What assumption did you make?

11. Gary takes four bus trips on each day of the weekend. He spends $16 each weekend on bus fares. How much does each trip cost?

12. Copy the pattern.

$(-12) \div (-3) =$ ■
$(-9) \div (-3) =$ ■
$(-6) \div (-3) =$ ■
$(-3) \div (-3) =$ ■
$0 \div (-3) =$ ■
$(+3) \div (-3) =$ ■
$(+6) \div (-3) =$ ■

a) Use integer chips to complete the first four lines. Describe the pattern.

b) Extend the pattern to determine the quotient $(+6) \div (-3)$.

13. Copy the pattern.

$(-8) \div (-2) =$ ■
$(-6) \div (-2) =$ ■
$(-4) \div (-2) =$ ■
$(-2) \div (-2) =$ ■
$0 \div (-2) =$ ■
$(+2) \div (-2) =$ ■
$(+4) \div (-2) =$ ■

a) Use integer chips to complete the first four lines. Describe the pattern.

b) Extend the pattern to determine the quotient $(+4) \div (-2)$.

14. The deepest recorded dive is 500 m for an emperor penguin and 2000 m for a sperm whale.

a) Use the division of two integers to represent how many times as deep a sperm whale can dive as an emperor penguin.

b) How can you model the division using only 20 integer chips?

c) What is the quotient?

Extend

15. Divide each of the following using integer chips or diagrams of chips. Explain your reasoning.

a) $(+15) \div (+5) \div (+3)$

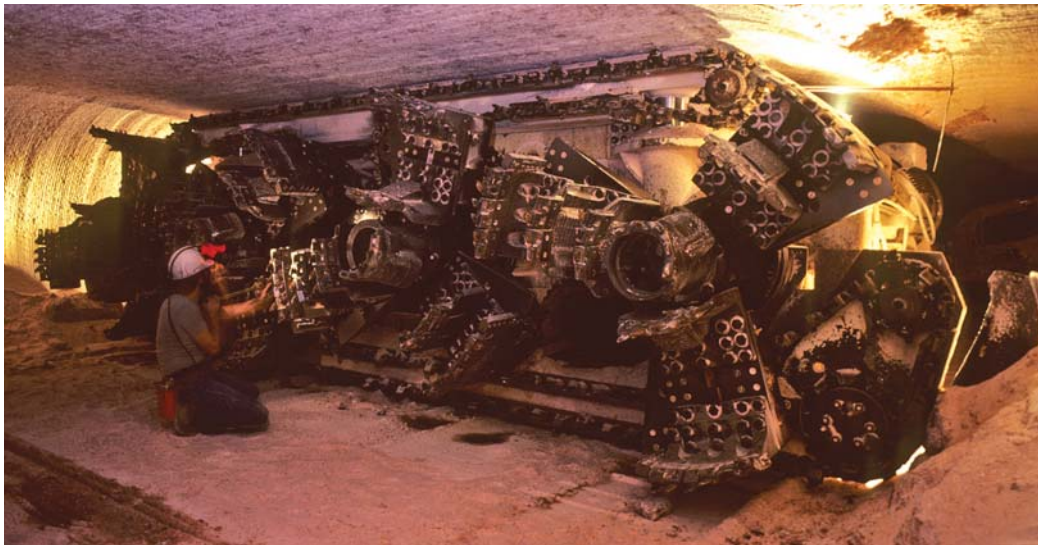
b) $(-24) \div (-2) \div (+4)$

c) $(-20) \div (+2) \div (-5)$

d) $(-18) \div (+2) \div (+3)$

16. Since sunset 6 h ago, the temperature in Brandon, Manitoba, has decreased from +1 °C to −11 °C. Predict what the temperature will be 3 h from now. What assumptions did you make?

8.4 Dividing Integers



Focus on...

After this lesson, you will be able to...

- determine integer quotients using a number line
- apply a sign rule when dividing integers

Farmers around the world use fertilizers made from potash mined in Saskatchewan. The province produces over 40% of the world's supply of potash.

To reach the potash, miners are lowered down a vertical mineshaft in a cage. Typical mineshafts are 900 m to 1100 m deep. The cage descends at about 6 m/s. How could you use integer chips to determine the time it takes to descend 900 m? Describe any difficulty you see in using integer chips to determine the time.

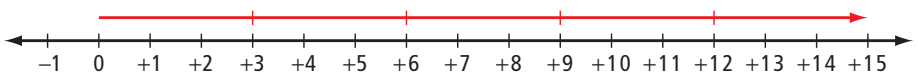
Explore the Math

Materials

- red and blue integer chips

How can you divide two integers?

1. The diagram shows how you can model the division $(+15) \div (+3)$ using a number line.



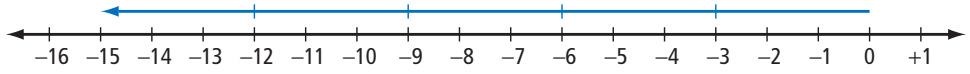
a) How are the two integers in the division $(+15) \div (+3)$ shown in the diagram?

b) Model $(+15) \div (+3)$ using integer chips. What is the quotient?

c) How does the number line show the quotient?

d) Explain how the diagram can also model the division $(+15) \div (+5)$.

2. The diagram shows how you can model the division $(-15) \div (-3)$ using a number line.



a) How are the two integers in the division $(-15) \div (-3)$ shown in the diagram?

b) Model $(-15) \div (-3)$ using integer chips. What is the quotient?

c) How does the number line show the quotient?

d) Explain how the diagram can also model the division $(-15) \div (+5)$.

3. a) Model the division $(-15) \div (+3)$ using a number line. Explain your reasoning.

b) Copy and complete the division statement $(-15) \div (+3) =$ ■.

c) Explain how your diagram can also model the division $(-15) \div (-5)$.

4. Can you use the same methods as in #1 to #3 to model the division $(+15) \div (-3)$? Explain.

5. The first row of the table shows a multiplication statement and the two division statements related to it. Copy and complete the table.

Multiplication Statement	Related Division Statements	
$(+2) \times (+4) = +8$	$(+8) \div (+4) = +2$	$(+8) \div (+2) = +4$
$(+6) \times (+2) = +12$		
$(+3) \times (-5) = -15$		
$(-3) \times (+6) = -18$		
$(-5) \times (-4) = +20$		
$(-1) \times (-9) = +9$		

6. Copy each of the following statements. Use your results from the table to complete each statement using the word "positive" or the word "negative."

The quotient of two integers with the same sign is ■.
The quotient of two integers with different signs is ■.

Reflect on Your Findings

7. a) How can you use a number line to divide two integers? In your description, state any limitations of your method.

b) How can you use the signs of two integers to help determine their quotient?

Example 1: Divide Integers

Calculate.

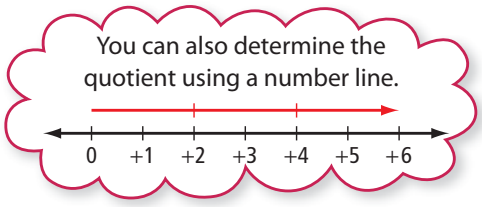
a) $(+6) \div (+2)$ **b)** $(-12) \div (-6)$
c) $(-20) \div (+4)$ **d)** $(+42) \div (-14)$

Solution

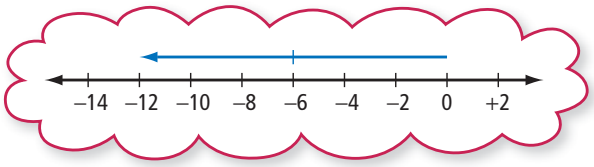
sign rule
(for division)
- the quotient of two integers with the same sign is positive
- the quotient of two integers with different signs is negative

Divide the numerals and then apply a **sign rule.**

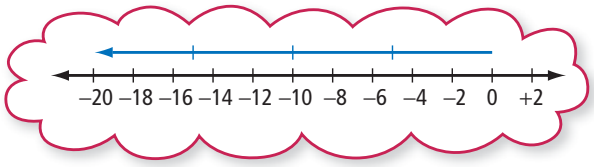
a) $6 \div 2 = 3$
The integers $+6$ and $+2$ have the same sign, so the quotient is positive.
$(+6) \div (+2) = +3$



b) $12 \div 6 = 2$
The integers -12 and -6 have the same sign, so the quotient is positive.
$(-12) \div (-6) = +2$



c) $20 \div 4 = 5$
The integers -20 and $+4$ have different signs, so the quotient is negative.
$(-20) \div (+4) = -5$



d) $42 \div 14 = 3$
The integers $+42$ and -14 have different signs, so the quotient is negative.
$(+42) \div (-14) = -3$

C 42 ÷ 14 +/− = -3.

Check:
$(-3) \times (-14) = +42$

You can use multiplication to check your division.

Tech Link

To enter a positive integer on your calculator, you do not need to enter the positive sign. You do need to enter the negative sign for a negative integer. On most calculators, the key used to enter a negative sign is not the subtraction key. Check that the key sequence shown in Example 1d) works correctly on your calculator. Modify the sequence, if necessary.

Show You Know

Calculate.

a) $(+24) \div (+8)$ **b)** $(+30) \div (-10)$
c) $(-48) \div (-12)$ **d)** $(-66) \div (+11)$

Example 2: Apply Integer Division

Daria and four of her friends went out for lunch. They agreed to split the cost equally. The total bill came to $85, which Daria paid on her credit card. How much did each of her friends owe Daria?

Solution

Use the division of two integers to represent the situation.
Represent the total cost of $85 by the integer -85.
Represent the 5 people by the integer $+5$.
Each person's share can be represented by the expression $(-85) \div (+5)$.
$(-85) \div (+5) = -17$
Each of her friends owed Daria $17.

85 ÷ 5 = 17. The integers −85 and +5 have different signs, so the quotient is negative.

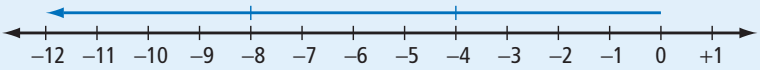
Check.
Use multiplication to check the division.
$(-17) \times (+5) = -85$

Show You Know

Pierre paid $42 to admit himself and two of his friends into a science museum. What was the cost of each admission?

Key Ideas

- You can model some integer divisions on a number line.



$(-12) \div (-4) = +3$ $(-12) \div (+3) = -4$

- You can divide two integers by dividing the numerals and applying the sign rules.
  - The quotient of two integers with the same sign is positive.
    $(+6) \div (+2) = +3$ $(-6) \div (-2) = +3$
  - The quotient of two integers with different signs is negative.
    $(+6) \div (-2) = -3$ $(-6) \div (+2) = -3$

Communicate the Ideas

1. To model the division $(+15) \div (+3)$ on a number line, you first draw an arrow that represents $+15$. You then have two choices:
- You can cut the arrow into parts that each represent $+3$ and count how many parts there are.
- You can cut the arrow into three equal parts and determine the value that each part represents.

Which choice do you prefer? Explain.

2. Aziza used a number line to model the division $(-12) \div (-2)$. Yuri used a number line to model the division $(-12) \div (+6)$. They drew the same diagram. What was the diagram?

3. Michel said, "When I divide $+6$ by $+3$, $+2$, or $+1$, the quotient is less than or equal to $+6$. If I divide -6 by $+3$, $+2$, or $+1$, I think the quotient should be less than or equal to -6." Do you agree with him? Explain.

4. Without doing any calculations, Stefani said that the quotients $(-252) \div (-18)$ and $(+252) \div (+18)$ must be the same. How did she know?

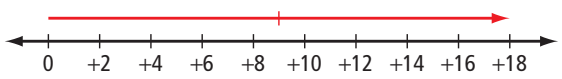
Check Your Understanding

Practise

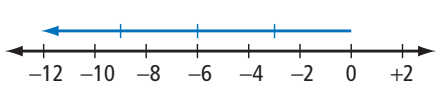
For help with #5 to #10, refer to Example 1 on page 308.

5. Write two division statements that each diagram could represent.

a)

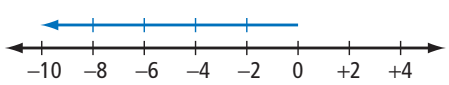


b)

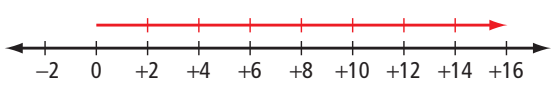


6. Write two division statements that each diagram could represent.

a)



b)



7. Determine each quotient using a number line.

a) $(+12) \div (+6)$ **b)** $(-20) \div (-4)$

c) $(-8) \div (+4)$ **d)** $(-10) \div (-5)$

8. Determine each quotient using a number line.

a) $(-14) \div (-7)$ **b)** $(+16) \div (+4)$

c) $(-22) \div (+2)$ **d)** $(-15) \div (-5)$

9. Calculate and check.

a) $(+20) \div (+5)$ **b)** $(+36) \div (-6)$

c) $(-57) \div (+19)$ **d)** $(-84) \div (-42)$

10. Calculate.

a) $(-26) \div (-26)$ **b)** $(+95) \div (-5)$

c) $0 \div (-33)$ **d)** $(-68) \div (+17)$

Apply

For help with #11 to #15, refer to Example 2 on page 309. Use the division of two integers to represent each situation and solve the problem.

11. Raoul borrowed \$15 per month from his mother to pay for the art supplies he needed for an evening class. At the end of the course, he owed his mother \$60. How long was the course?

12. a) A submarine took 16 min to dive 96 m from the surface. How far did it dive per minute?

b) The submarine took 12 min to climb back to the surface. How far did it climb per minute?

13. A scuba diver was collecting water samples from a lake. He collected samples at 5-m intervals starting at 5 m below the surface. He collected the final sample at a depth of 35 m. How many samples did he collect?

14. Mina was drilling down through a 21-cm thick concrete floor to install a new plumbing pipe. She drilled for 5 min, took a break, and then finished drilling in another 2 min. At what rate did the drill cut through the floor, in centimetres per minute? What assumptions did you make?

15. A school spent $384 to buy a set of 32 calculators. What was the cost of each calculator?

16. Without evaluating the quotients, identify the quotient with the least value. Explain your reasoning.

$(+2408) \div (+43)$
$(-2408) \div (-43)$
$(+2408) \div (-43)$

17. If 28 times an integer is -448, what is the integer?

18. Copy and complete each statement.

a) $(+72) \div (\blacksquare) = +9$

b) $(\blacksquare) \div (+12) = -10$

c) $(\blacksquare) \div (-13) = -11$

d) $(-84) \div (\blacksquare) = +6$

19. Write a word problem that you can solve using the expression $(-80) \div (+16)$.

20. Create your own word problem that involves integer division. Make sure that you can solve your problem and that the calculation results in an integer. Give your problem to a classmate to solve.

Extend

21. Describe each pattern. Then write the next three terms in each pattern.

a) $+125\,000, +25\,000, +5000, +1000, \ldots$

b) $-512, +256, -128, +64, \ldots$

c) $-1\,000\,000, -100\,000, -10\,000, -1000, \ldots$

d) $+1458, -486, +162, -54, \ldots$

22. The sum of two integers is $+20$. Dividing the larger integer by the smaller integer gives a quotient of -3. What are the two integers?

MATH LINK

The temperature of still, dry air decreases by about 6 °C for each kilometre increase in altitude. On a still, dry day, the temperature in Yellowknife, Northwest Territories, was −11 °C. The air temperature outside a plane flying above Yellowknife was −53 °C.

a) Approximately how much lower was the temperature outside the aircraft than the temperature in Yellowknife?

b) How high was the aircraft above Yellowknife?

8.5

Applying Integer Operations

Focus on...

After this lesson, you will be able to...

- apply the order of operations to solve problems involving integers

A famous submersible, the *Ben Franklin*, is at the Vancouver Maritime Museum. This submersible was built to study the currents and sea life along the east coast of North America in the Gulf Stream.

The *Ben Franklin* explored the Gulf Stream at depths from 200 m to 600 m. How would you represent these depths with integers? What operation would you use to find how many times as great a depth of 600 m is as a depth of 200 m?

Did You Know?

The *Ben Franklin* is named after the American scientist and diplomat Benjamin Franklin, 1706–1790. While a passenger on transatlantic voyages, he spent his time measuring ocean temperatures. Franklin mapped a stream of warm water about 70 km wide flowing across the cold Atlantic. He named this stream the Gulf Stream.

Explore the Math

Why is it important to know the order of operations when solving problems involving integers?

Laura, Abeni, and Rob were discussing the following problem.

A submersible dives from the surface at 15 m/min for 6 min and then at 25 m/min for 20 min. What is the depth of the submersible after the dive?

They worked together to write the following expression to solve the problem.

$6 \times (-15) + 20 \times (-25)$

Then they evaluated the expression independently.
Laura evaluated the expression as $+1750$.
Abeni evaluated the expression as -590.
Rob evaluated the expression as -750.

1. Explain how the expression represents the problem.

2. Which student evaluated the expression correctly? How do you know?

3. What errors did the other two students make?

4. What is the depth of the submersible after the dive?

Reflect on Your Findings

5. Why is it important to know the order of operations when solving problems involving integers?

WWW Web Link

Submersibles are still used to explore the world's oceans. The ROPOS submersible operates out of Sidney, British Columbia. ROPOS can reach a depth of 5000 m.

To learn more about the *Ben Franklin* and ROPOS submersibles, go to www.mathlinks8.ca and follow the links.

Literacy Link

Omitting Positive Signs or Brackets

A positive integer can be written without the positive sign or brackets. For example, $(+3) \times (+4)$ can be written as 3×4.

Negative integers must include the negative sign. The brackets can be omitted from a negative integer that does not follow an operation symbol. For example, $(-9) \div (-3)$ can be written as $-9 \div (-3)$.

Example 1: Use the Order of Operations

Calculate.

a) $(-15) \div (-3) - (+4) \times (-2)$

b) $(-6) - (-9) + (-14) \div (+2)$

c) $-8 + (-2) \times [4 + (-1)]$

Solution

a)
$(-15) \div (-3) - (+4) \times (-2)$ Multiply and divide in order, from left to right.
$= (+5) - (+4) \times (-2)$
$= (+5) - (-8)$ Subtract.
$= +13$

b)
$(-6) - (-9) + (-14) \div (+2)$ Divide.
$= (-6) - (-9) + (-7)$ Add and subtract in order, from left to right.
$= (+3) + (-7)$
$= -4$

c)
$-8 + (-2) \times [4 + (-1)]$ Brackets.
$= -8 + (-2) \times 3$ Multiply.
$= -8 + (-6)$ Add.
$= -14$

Grouping is shown using square brackets, because -1 is already in round brackets.

Show You Know

Calculate.

a) $(+4) + (-7) \times (-3) - (+5)$

b) $(-16) \div [(+5) - (+6) + (-7)]$

c) $-2 \times [5 + (-3)] + (-15)$

Did You Know?

Peguis is about 190 km north of Winnipeg and is the largest First Nations community in Manitoba.

Example 2: Apply Integer Operations

One week in March in Peguis, Manitoba, the daily high temperatures were −2 °C, −6 °C, +1 °C, +2 °C, −5 °C, −8 °C, and +4 °C. What was the mean daily high temperature for that week?

Solution

The mean temperature is the sum of the temperatures divided by the number of temperatures.

$$\frac{(-2)+(-6)+(+1)+(+2)+(-5)+(-8)+(+4)}{7}$$

The division bar is a division and grouping symbol, so you can rewrite the numerator inside brackets.

$= [(-2) + (-6) + (+1) + (+2) + (-5) + (-8) + (+4)] \div 7$ Brackets.

$= -14 \div 7$ Divide.

$= -2$

You can omit the round brackets from both integers and the positive sign from +7.

The mean daily high temperature for that week was −2 °C.

Literacy Link

Understanding the Mean

The *mean* of a set of integers is found by dividing their sum by the number of integers. For example, the mean of −4, +8, and −10 is $\frac{(-4)+(+8)+(-10)}{3}$, which equals −2.

Show You Know

On four days in June in Resolute, Nunavut, the daily low temperatures were −6 °C, 0 °C, +1 °C, and −7 °C. What was the mean daily low temperature for those four days?

Key Ideas

- When solving a problem, you need to decide which operation(s) to perform on integers.
- Some integer problems involve the order of operations.
- The order of operations for integers is the same as for whole numbers and decimals.
  - Brackets.
  - Multiply and divide in order, from left to right.
  - Add and subtract in order, from left to right.

Communicate the Ideas

1. Lance evaluated the expression $-2 \times (4 + 5) + 3$ to equal 0.

a) What mistake did he make?

b) What is the correct value of the expression?

2. Ivan said that the mean of −18, −16, +11, and +15 is positive. Without calculating the value of the mean, Norah disagreed with him. How did she know that the mean is not positive?

3. If 15 times an integer is −255, would it be easier to determine the integer using multiplication or division? Explain.

Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 313.

4. Calculate using the order of operations.

a) $(+30) \div (-10) + (-20) \div (-1)$

b) $(-2) \times [(+10) - (+8)] + (-7)$

c) $(+6) + (+9) \times (-5) \div (-3)$

5. Calculate using the order of operations.

a) $(-4) - (+8) \times (-2) - (+15)$

b) $(-3) + (-18) \div (+2) \div (-3)$

c) $(+16) \div [(+4) - (+2)] + (-4)$

6. Calculate.

a) $(4 - 7) \times 2 + 12$

b) $-10 \div 5 + 3 \times (-4)$

c) $3 \times [14 + (-18)] - 8 \div (-4)$

7. Calculate.

a) $-16 \div 2 \times (3 + 1)$

b) $5 + (-9) \times 4 \div (-1)$

c) $25 + (-10) - 3 \times [2 - (-2)]$

Apply

For help with #8 to #12, refer to Example 2 on page 314.

8. The daily low temperatures in Prince Rupert, British Columbia, for five days in January were −4 °C, +1 °C, −2 °C, +1 °C, and −6 °C. What is the mean of these temperatures?

9. The table shows changes in the number of subscribers to a community newsletter over a six-month period.

Month	Change in the Number of Subscribers
1	+8
2	+6
3	−12
4	+5
5	−9
6	−10

a) What was the mean change per month in the number of subscribers?

b) There were 207 subscribers at the beginning of this period. How many were there at the end?

10. Over a ten-year period, the population of Saskatchewan decreased from 989 000 to 979 000. What was the mean population change per year?

11. The mean of five integers is −11. What is the sum of the integers?

12. A golfer had a mean score of −3 for the 4 rounds of a golf tournament.

a) What was the golfer's score for the whole tournament?

b) If par for the course is 72 strokes, how many strokes did the golfer take to complete the tournament?

Sports Link

Golf Scores

Par for a golf course is the total number of strokes an expert golfer should take to complete the course. An integer shows a golfer's performance for each round of golf. Using 68 strokes to complete a round on a par 70 course gives a score of 2 below par or −2 for that round. Using 74 strokes to complete a round on a par 70 course gives a score of 4 over par or +4 for that round.

13. The average temperature of Earth's surface is about 15 °C. The temperature of Earth's crust increases by about 25 °C for each kilometre below the surface. What is the average temperature 3 km below Earth's surface?

Did You Know?

The high temperatures below Earth's surface can create hot springs. Water is heated underground and then runs to the surface before it can cool down. The water that feeds the Miette Hot Springs in Jasper National Park is at 54 °C when it reaches the surface. Hot springs support many forms of life. Dr. Kathleen Londry of the University of Manitoba studies an endangered species of snail that lives in the Cave and Basin Hot Springs in Banff National Park.

14. Ahmed had $100 in his savings account at the start of his summer job. For the next eight weeks, he added $70 to his savings each week. After he went back to school, he withdrew $55 per week from his savings. For how many weeks did he make withdrawals until his savings were gone?

15. A new freezer is at a room temperature of 22 °C. When the freezer is turned on, the temperature inside drops by 4 °C per hour. How long does it take the freezer to reach −10 °C?

16. A hang glider descends at 50 m/min for 3 min. The glider then catches an updraft and rises at 100 m/min for 2 min.

a) What is the overall change in the hang glider's altitude over this 5-min period?

b) What is the mean rate of change in the altitude over this period?

17. Because the batteries are low, Darren's watch is losing 9 min every hour. At 8:00 p.m., he sets the watch correctly. What time will his watch show when the correct time is 10:00 a.m. the next day?

18. a) A store that sells ski equipment lost a total of $18 000 in June, July, and August. What was the mean loss per month?

b) The store broke even in April, May, and September. The store owner wants to make a profit of $54 000 for the year. To meet this target, what mean profit per month does the store need to make in the other six months?

19. Rohana earns $50 a week from babysitting. She spends $25, saves $15, and uses the rest to repay a loan of $100 from her sister.

a) After six weeks, how much has Rohana spent, how much has she saved, and how much does she still owe her sister?

b) How many more weeks will Rohana take to pay off the loan?

20. Write and evaluate an expression that represents each statement.

a) Subtract the product of 3 and −8 from 20.

b) Add the product of 4 and 5 to the product of −2 and −3.

c) Divide −62 by the sum of −11 and 9.

d) Multiply the sum of −3 and −5 by 3, then divide by −4 and subtract 13.

Extend

21. Copy each statement. Complete it by including operation symbols.

a) $2 \blacksquare 3 \blacksquare 4 \blacksquare 5 = -14$

b) $3 \blacksquare [14 \blacksquare (-2)] \blacksquare 30 = 6$

22. The mean of two integers is −17. The product of the two integers is 273. What are the two integers?

23. A multiple-choice test with 50 questions has five possible choices for each question. There are 4 marks for each correct answer, −1 mark for each incorrect answer, and 0 marks for each unanswered question.

a) What is the total score of a student with 35 correct answers, 10 incorrect answers, and 5 unanswered questions?

b) Express the student's score as a percent.

24. Here is one way of using four −2s and the order of operations to write an expression that equals 1.

$(-2) \div (-2) + (-2) - (-2)$

Use four −2s and the order of operations to write expressions that equal 2, 3, 4, 5, 6, and 8.

Sports Link

Scoring System in the Modern Pentathlon

Monica Pinette from Langley, British Columbia, is a successful Canadian athlete. She has won gold medals at both the Canadian and the Pan American championships in the modern pentathlon. This sport includes five events: shooting, fencing, swimming, show jumping, and running. The events are all held on the same day. The winner is decided using a points system that involves integer operations.

The running event is a 3000-m cross-country race. A male athlete scores 1000 points for finishing this event in 10-min. A female athlete scores 1000 points for finishing in 11 min 20 s. Each whole second below these times is worth 4 extra points. Each whole second above these times results in a 4-point deduction.

1. Show how the following points are calculated.

a) 920 points for a male with a time of 10 min 20 s

b) 1060 points for a female with a time of 11 min 5 s

2. Calculate the points earned by each of the following athletes.

a) a female with a time of 11 min 43 s

b) a male with a time of 9 min 51 s

3. Calculate the time taken by each of the following athletes.

a) a female who scores 1100 points

b) a male who scores 892 points

Did You Know?

Another sport that includes several events is the decathlon. Dave Steen from New Westminster, British Columbia, won a bronze medal in the decathlon at the summer Olympics in Seoul, South Korea. He competed in ten events over two days.

WWW Web Link

To learn more about the modern pentathlon, including the scoring of the other four events, go to www.mathlinks8.ca and follow the links.