

6.1

Multiplying a Fraction and a Whole Number

Focus on...

After this lesson, you will be able to...

- multiply a fraction and a whole number
- solve problems involving the multiplication of a fraction and a whole number

Chess is one of the most popular board games. It has been estimated that about $\frac{1}{5}$ of adult Canadians play chess at least once a year. The chess board shown has black and white squares. What fraction of the total number of squares are black? If you were told the total area of all the squares, how could you determine the total area of the black squares?



Explore the Math

How can you model the multiplication of a fraction and a whole number?

1. a) How do the pattern blocks model an addition? Describe it.



If a yellow hexagon represents one whole, what do a red trapezoid, a blue rhombus, and a green triangle represent?



- b) How do the pattern blocks also model a multiplication? Describe it.
- c) Work with a partner to explore other manipulatives you could use to model the multiplication.
2. a) Work with a partner to explore how you could use diagrams to model $4 \times \frac{1}{6}$.
- b) Write an equation to represent your model.

Materials

- pattern blocks

Literacy Link

Understanding Multiplication

The product of 4 and 2 is 8, because $4 \times 2 = 8$.

The equation $4 \times 2 = 8$ means that 4 groups of 2 make 8. You can also think of 4×2 as the repeated addition $2 + 2 + 2 + 2$.

3. a) Model $2 \times \frac{4}{3}$ using the method of your choice.
 b) Write an equation to represent your model.

Reflect on Your Findings

4. a) Share your models with your classmates.
 b) Suggest other manipulatives or diagrams you could use. How would you use them?

Example 1: Multiply Using A Model

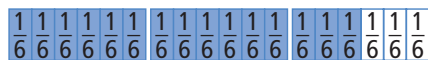
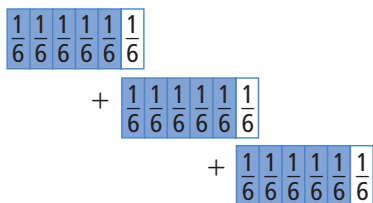
Determine $3 \times \frac{5}{6}$. Express the product in lowest terms.

Solution

You can express the multiplication as a repeated addition.

$$3 \times \frac{5}{6} = \frac{5}{6} + \frac{5}{6} + \frac{5}{6}$$

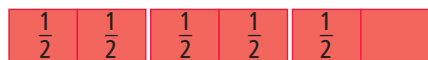
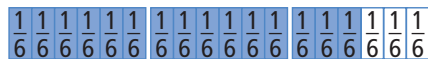
Model the fractions using fraction strips.



Count the shaded parts of the strips.

$$\frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{15}{6}$$

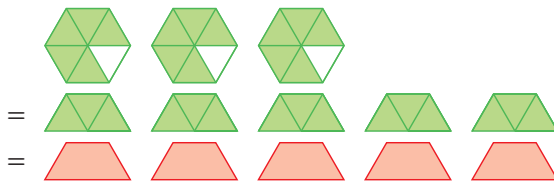
Write the product in lowest terms.



$$\frac{15}{6} = \frac{5}{2}$$

$$\text{So, } 3 \times \frac{5}{6} = \frac{5}{2}$$

You could model with pattern blocks instead of fraction strips.



You could divide the numerator and denominator by a common factor.

$$\begin{array}{c} \div 3 \\ \frac{15}{6} = \frac{5}{2} \\ \div 3 \end{array}$$

Literacy Link

Classifying Fractions

In a *proper fraction*, such as $\frac{1}{2}$ or $\frac{5}{6}$, the denominator is greater than the numerator.

In an *improper fraction*, such as $\frac{5}{2}$ or $\frac{4}{3}$, the numerator is greater than the denominator.

A *mixed number*, such as $1\frac{1}{4}$ or $4\frac{3}{5}$, includes a whole number and a proper fraction.

Strategies

Model It

The product of a natural number and a proper fraction is less than the natural number.

$$\frac{5}{2} < 3$$

Show You Know

Determine each product using models. Express the product in lowest terms.

a) $2 \times \frac{5}{6}$ b) $4 \times \frac{2}{3}$

Example 2: Multiply Using a Diagram

Determine $3 \times \frac{2}{5}$. Express the product in lowest terms.

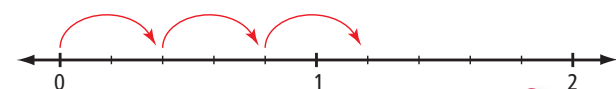
Strategies

Draw a Diagram

Solution

$$3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$$

Model the fractions using a number line.

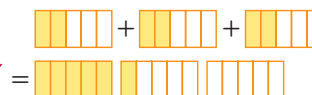


$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}$$

The answer is already in lowest terms.

So, $3 \times \frac{2}{5} = \frac{6}{5}$.

You could draw rectangles instead of using a number line.



Show You Know

Determine each product using a diagram. Express the product in lowest terms.

a) $2 \times \frac{3}{2}$ b) $4 \times \frac{5}{8}$

Example 3: Apply Multiplication With Fractions

A spider has eight legs. An ant has $\frac{3}{4}$ as many legs as a spider. How many legs does an ant have?

Solution

An ant has $\frac{3}{4}$ of the number of legs that a spider has.

$$\frac{3}{4} \text{ of } 8 \text{ means } \frac{3}{4} \times 8.$$

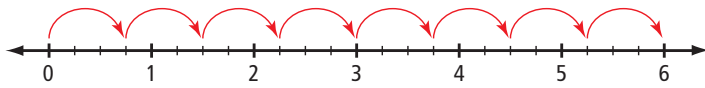
Literacy Link

In mathematics, the word *of* often indicates multiplication.

Multiplying $\frac{3}{4} \times 8$ gives the same answer as multiplying $8 \times \frac{3}{4}$.

Determine $8 \times \frac{3}{4}$.

Model the multiplication as a repeated addition on a number line.



The result is 6. So, $8 \times \frac{3}{4} = 6$.

An ant has six legs.

An ant has $\frac{3}{4}$ as many legs as a spider, so an ant has fewer legs than a spider.
 $6 < 8$



Literacy Link

Commutative Property

The *commutative property* states that $a \times b = b \times a$.

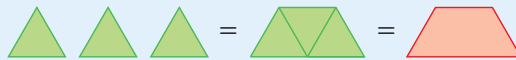
Show You Know

Jenelle is making a recipe that calls for six scoops of flour. She wants to make only $\frac{2}{3}$ of the recipe. How many scoops will she need to use?

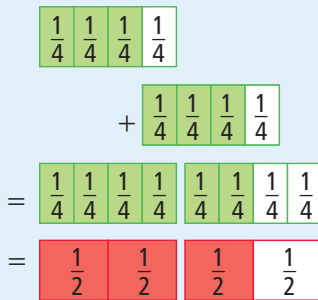
Key Ideas

- You can show the multiplication of a fraction and a whole number using models and diagrams.

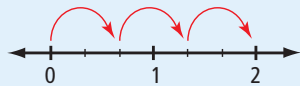
$$3 \times \frac{1}{6} = \frac{1}{2}$$



$$2 \times \frac{3}{4} = \frac{3}{2}$$



$$3 \times \frac{2}{3} = 2$$



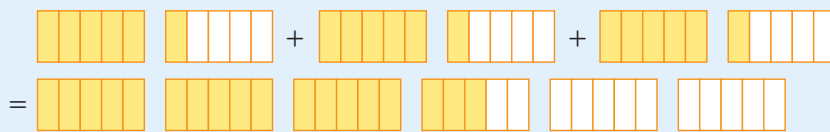
- Multiplying a fraction and a whole number in either order gives the same result.

$$10 \times \frac{2}{5} = 4$$

$$\frac{2}{5} \times 10 = 4$$

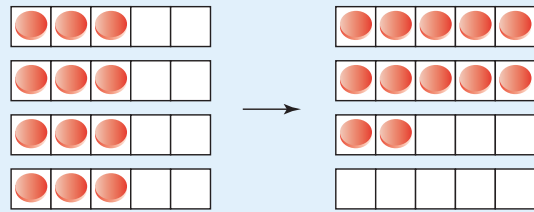
Communicate the Ideas

1. The diagram models $3 \times \frac{6}{5}$.



- a) What equation does the diagram represent?
 b) If a hexagon represents one whole, could you use pattern blocks to model the same multiplication? Explain.

2. Makoto found his own way to model $4 \times \frac{3}{5}$ by using counters on grids.



- a) Why did he use 5-by-1 grids?
 b) Why did he use four grids?
 c) How does Makoto's model show the product?

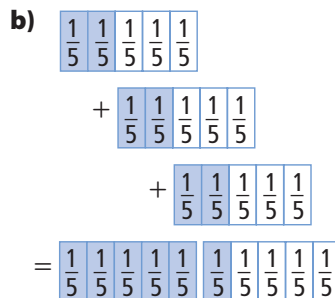
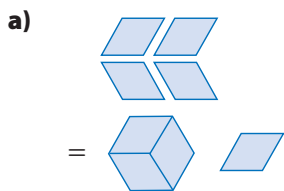
3. Nadine said that she had her own method for determining $4 \times \frac{3}{5}$. She first multiplied 4 and 3 to get 12. She then wrote the product as $\frac{12}{5}$. Do you agree with Nadine's method for multiplying a whole number and a fraction? Explain using other examples.

Check Your Understanding

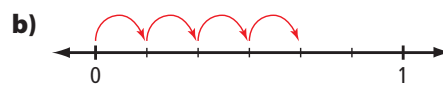
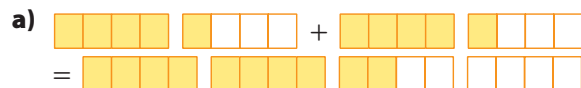
Practise

For help with #4 to #7, refer to Examples 1 and 2 on pages 199–200.

4. What equation does each model represent? For pattern blocks, assume that a hexagon represents one whole.



5. What equation does each diagram represent?



6. Determine each product using manipulatives or diagrams.

- a) $4 \times \frac{1}{2}$ b) $3 \times \frac{7}{10}$
 c) $5 \times \frac{2}{3}$ d) $3 \times \frac{3}{8}$

7. Determine each product.

a) $3 \times \frac{1}{8}$

b) $6 \times \frac{1}{4}$

c) $2 \times \frac{6}{5}$

d) $2 \times \frac{4}{3}$

Apply

For help with #8 to #9, refer to Example 3 on page 200.

8. The width of a Canadian flag is $\frac{1}{2}$ of its length. What is the width of a Canadian flag that is 4 m long?



9. A minibus that seats 12 people is $\frac{3}{4}$ full. How many people are seated in the minibus?

10. a) What fraction of the surface area of a cube is the area of one face?

b) What is the area of each face of a cube of surface area 6 cm^2 ?

11. Ron's car uses 12 L of gasoline per 100 km of highway driving. Asma's car uses only $\frac{5}{6}$ as much fuel. How much fuel does Asma's car use per 100 km of highway driving?

12. Nunavut covers about $\frac{1}{5}$ of the area of Canada. The area of Canada is about ten million square kilometres. What is the approximate area of Nunavut?

13. Suppose a friend knows how to multiply whole numbers, but not fractions.

a) How could you use the following pattern to show your friend how to calculate $\frac{1}{2} \times 10$?

$$4 \times 10 = 40$$

$$2 \times 10 = 20$$

$$1 \times 10 = 10$$

$$\frac{1}{2} \times 10 = \blacksquare$$

b) Make up a pattern to show your friend how to calculate $\frac{1}{3} \times 9$.

14. Write a word problem that you can solve using the expression $\frac{1}{4} \times 8$.

Extend

15. There are 30 students in a class. Four fifths of them have brown eyes. How many students have brown eyes?

16. The perimeter of an isosceles triangle is 15 cm. The shortest side equals $\frac{1}{5}$ of the perimeter. What are the side lengths of the triangle?

17. A ball dropped to the ground bounces back to $\frac{2}{3}$ of its previous height. If the ball is dropped straight down from a height of 81 cm, how far does it travel altogether by the time it hits the ground for the fifth time?

MATH LINK

A quarter of Canada's 20 ecozones are marine ecozones, which include parts of oceans. The rest of Canada's ecozones are terrestrial ecozones. They include parts of the land, and may contain rivers, lakes, and wetlands.

a) How many marine ecozones does Canada have?

b) How many terrestrial ecozones does Canada have?

6.2

Dividing a Fraction by a Whole Number

Focus on...

After this lesson, you will be able to...

- divide a fraction by a whole number
- solve problems involving the division of fractions by whole numbers

Iqaluit, the capital of Nunavut, has frost on about $\frac{3}{4}$ of the days in a year. Iqaluit has frost on five times as many days as Vancouver, British Columbia. Work with a partner to explore how you might determine the fraction of the days in a year that Vancouver has frost.



Materials

- pattern blocks
- fraction strips

Literacy Link

Understanding Division

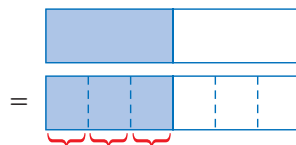
In the equation $6 \div 2 = 3$, the dividend is 6, the divisor is 2, and the quotient is 3.

The equation $6 \div 2 = 3$ means that in 6 there are 3 groups of 2. This division statement also means that if 6 is separated into 2 equal groups, there are 3 in each group.

Explore the Math

How can you model the division of a fraction by a whole number?

1. **a)** The long rectangle in the following diagram represents one whole. The diagram models a division. Describe it.



- b)** Work with a partner to explore other diagrams you could use to model the division.
2. **a)** Work with a partner to explore how you could use manipulatives to model $\frac{2}{3} \div 2$.
 - b)** Write an equation to represent your model.

Reflect on Your Findings

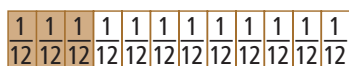
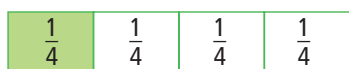
3. a) Share your models with your classmates.
- b) Can you think of other manipulatives or diagrams you could use? If so, explain how you would use them.

Example 1: Divide Using a Model

Determine $\frac{1}{4} \div 3$.

Solution

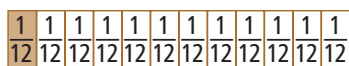
Use a fraction strip to represent $\frac{1}{4}$.



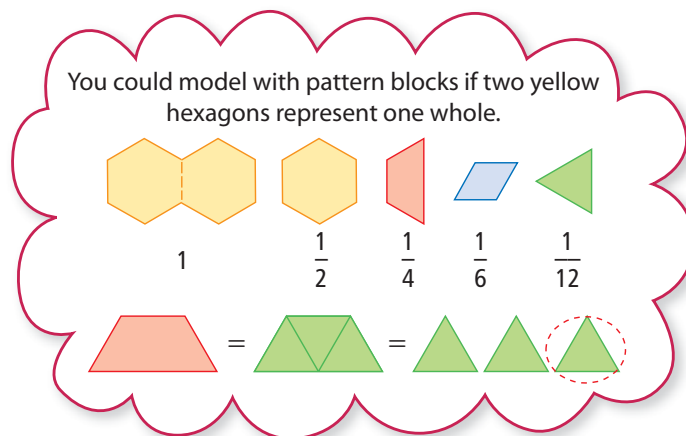
Identify the fraction strip that shows $\frac{1}{4}$ cut into three equal parts.

The fraction strip shows that $\frac{1}{4}$ is equivalent to $\frac{3}{12}$.

Each of the three equal parts of $\frac{1}{4}$ is $\frac{1}{12}$.



$$\frac{1}{4} \div 3 = \frac{1}{12}$$



M E

If you divide a fraction by a natural number greater than 1, the quotient is less than the original fraction.

$$\frac{1}{12} < \frac{1}{4}$$

Show You Know

Determine each quotient using models.

- a) $\frac{3}{4} \div 3$ b) $\frac{5}{6} \div 2$

Example 2: Divide Using Diagrams

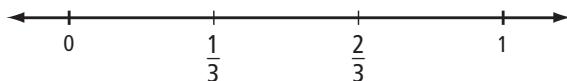
Determine $\frac{2}{3} \div 4$. Express the quotient in lowest terms.

Strategies

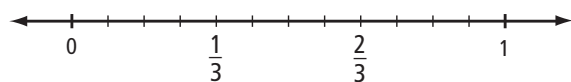
Draw a Diagram

Solution

Draw and label a number line that shows thirds.

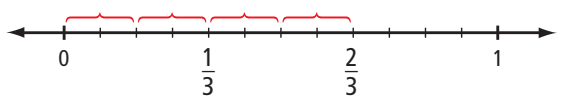


To model division by 4, cut each third into four equal parts.



There are 12 parts in the whole, so each part is $\frac{1}{12}$.

Use brackets to cut $\frac{2}{3}$ into four equal parts.

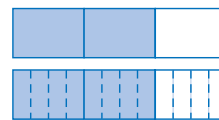


Each of the four parts is $\frac{2}{12}$.

$$\frac{2}{12} = \frac{1}{6}$$

$$\text{So, } \frac{2}{3} \div 4 = \frac{1}{6}.$$

You could draw a rectangle instead of a number line.



$$\text{So } \frac{2}{3} \div 4 = \frac{2}{12}$$

Show You Know

Determine each quotient using a diagram. Express the quotient in lowest terms.

a) $\frac{1}{2} \div 5$ b) $\frac{3}{5} \div 3$

Example 3: Apply Division With Fractions

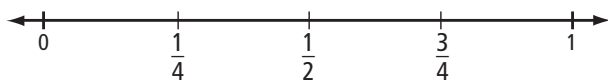
Mustafa used $\frac{3}{4}$ of a jar of pasta sauce on six servings of pasta. He used the same amount of sauce on each serving. What fraction of the jar of pasta sauce did he use on each serving?

Solution

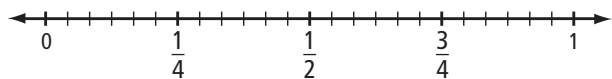
Determine $\frac{3}{4} \div 6$.

The $\frac{3}{4}$ of a jar is shared equally among six servings.

Draw and label a number line that shows quarters.

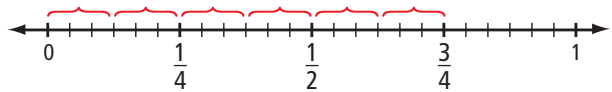


To model division by 6, cut each quarter into six equal parts.



There are 24 parts in the whole.
Each part is $\frac{1}{24}$.

Use brackets to cut $\frac{3}{4}$ into six equal parts.



Each of the six parts is $\frac{3}{24}$.

So, $\frac{3}{4} \div 6 = \frac{3}{24}$ or $\frac{1}{8}$

Mustafa used $\frac{1}{8}$ of a jar of pasta sauce on each serving.

M E

If $\frac{3}{4}$ of a jar is shared among six servings, the fraction of the jar for each serving must be less than $\frac{3}{4}$.

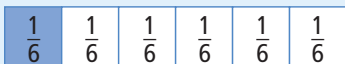
$$\frac{1}{8} < \frac{3}{4}$$

Show You Know

Four students equally shared $\frac{1}{2}$ of a cake. What fraction of the cake did each student eat?

Key Ideas

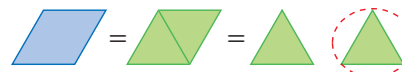
- You can show the division of a fraction by a whole number using models and diagrams.



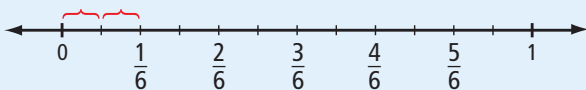
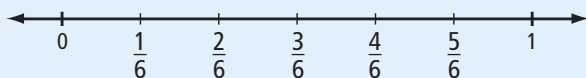
Each of the two equal parts of $\frac{1}{6}$ is $\frac{1}{12}$.



You could model with pattern blocks. If two hexagons represent one whole, then one rhombus represents one sixth.

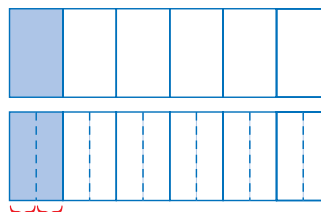


$$\frac{1}{6} \div 2 = \frac{1}{12}$$



$$\frac{1}{6} \div 2 = \frac{1}{12}$$

You could draw a rectangle instead of a number line.



Communicate the Ideas

1. Lana decided to model the division $\frac{2}{3} \div 3$ using a fraction strip divided into sixths. Could you use this fraction strip to solve the problem? Explain.
2.
 - a) If you use four hexagons to represent one whole, show how you can model $\frac{3}{4} \div 6$ using pattern blocks.
 - b) Can you model $\frac{3}{4} \div 6$ by using two hexagons to represent one whole? Explain.
3.
 - a) Model the division $\frac{1}{2} \div 2$ using manipulatives or diagrams.
 - b) Which method did you choose? Explain why you chose it.

Check Your Understanding

Practise

For help with #4 and #5, refer to Examples 1 and 2 on pages 205–206.

4. Determine each quotient using manipulatives or diagrams.
 - a) $\frac{1}{4} \div 2$
 - b) $\frac{1}{3} \div 3$
 - c) $\frac{1}{5} \div 2$
 - d) $\frac{5}{6} \div 4$
5. Determine each quotient.
 - a) $\frac{3}{5} \div 2$
 - b) $\frac{1}{5} \div 3$
 - c) $\frac{1}{2} \div 4$
 - d) $\frac{2}{3} \div 6$

Apply

For help with #6 to #8, refer to Example 3 on pages 206–207.

6. Two different South Indian fish curries, called dhopa and molee curry, both include coconut.
 - a) Dhopa requires $\frac{1}{2}$ a coconut to make two servings. What fraction of a coconut is in each serving?
 - b) Molee curry requires $\frac{1}{2}$ a coconut to make four servings. What fraction of a coconut is in each serving?
7. A pitcher of orange juice is $\frac{2}{3}$ full. If four students equally share the juice, what fraction of the full pitcher does each student get?

8. The areas of Alberta, Saskatchewan, and Manitoba are approximately equal. The sum of their areas is about $\frac{1}{5}$ of the area of Canada. Express the area of each of these provinces as a fraction of the area of Canada.
9. Ingrid runs three laps of a track in $\frac{1}{4}$ h.
On average, how much time does she take to run one lap? Express your answer
- as a fraction of an hour
 - in minutes



10. Mark uses $\frac{1}{3}$ of a tank of gasoline in a five-day work week driving to work and back. On average, what fraction of a tank does he use for each round trip?

11. Iqaluit has frost on about $\frac{3}{4}$ of the days in a year. It has frost on five times as many days as Vancouver. On what fraction of the days of the year does Vancouver have frost?
12. It takes $\frac{4}{5}$ of a roll of ribbon to wrap six packages. What fraction of a roll does it take to wrap three packages?
13. Create your own word problem that involves the division of a proper fraction by a whole number. Make sure that you can solve your problem. Give your problem to a classmate to solve.

Extend

14. Two fractions are equally spaced between $\frac{2}{5}$ and $\frac{4}{5}$. Determine the two fractions.



15. a) Model the division $\frac{2}{3} \div 4 = \frac{1}{6}$ using manipulatives or diagrams.
b) Explain how your method shows that $\frac{2}{3} \div \frac{1}{6} = 4$.

MATH LINK

The Montane Cordillera and Boreal Cordillera ecozones have approximately equal areas. The total area of these two ecozones equals about $\frac{1}{10}$ of the area of Canada. What fraction of the area of Canada does each of these ecozones cover?



6.3

Multiplying Proper Fractions

Focus on...

After this lesson, you will be able to...

- multiply two proper fractions
- solve problems involving the multiplication of two proper fractions



A two-toed sloth sleeps for 20 h per day. A chimpanzee sleeps $\frac{1}{2}$ that much. A horse sleeps $\frac{1}{2}$ as much as a chimpanzee. In a day, what fraction of the time that a two-toed sloth sleeps does a horse sleep? How do you know?

Explore the Math

Materials

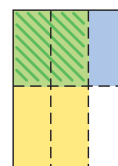
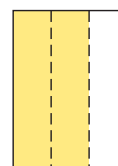
- six sheets of plain paper
- yellow and blue coloured pencils or crayons

How can you multiply two proper fractions?

1. You can determine $\frac{1}{2}$ of $\frac{2}{3}$ using paper folding.
 - Fold a rectangular piece of paper into thirds along its length. Open the paper and shade $\frac{2}{3}$ of it yellow.
 - Fold the paper in half across its width. Open the paper and shade half of it blue.

How does the model show $\frac{1}{2}$ of $\frac{2}{3}$?

Will the product be greater than or less than $\frac{2}{3}$?



2. Copy the table. Complete the table by using the method from #1. Do not write the products in lowest terms.

Multiplication	Product
$\frac{1}{2} \times \frac{2}{3}$	
$\frac{1}{3} \times \frac{1}{2}$	
$\frac{3}{4} \times \frac{1}{3}$	
$\frac{3}{4} \times \frac{2}{3}$	
$\frac{3}{4} \times \frac{3}{4}$	

3. For each row of the table, describe the relationship between
 a) the numerators b) the denominators
4. Use your answers from #3 to write a rule for multiplying two proper fractions.
5. a) Test your rule on the product $\frac{3}{4} \times \frac{1}{2}$.
 b) Check your answer to part a) by paper folding.



Reflect on Your Findings

6. a) How can you multiply two proper fractions by paper folding or using a rule?
 b) Which method do you prefer? Explain.

Example 1: Multiply Using Paper Folding

Determine $\frac{1}{2} \times \frac{3}{5}$.

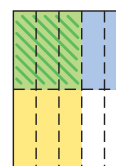
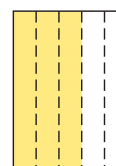
Solution

Fold a rectangular piece of paper into fifths along its length.
 Open the paper and shade three fifths of it yellow.
 Fold the paper in half across its width.

Open the paper and shade half of it blue.
 The folds make ten equal rectangles.
 Three of them are shaded both yellow and blue, so they appear green.

$$\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

The numerator in the product is the number of green rectangles.
 The denominator in the product is the total number of equal rectangles.



Show You Know

Determine each product using paper folding.

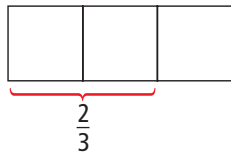
a) $\frac{1}{4} \times \frac{1}{2}$ b) $\frac{2}{3} \times \frac{2}{3}$

Example 2: Multiply Using Diagrams

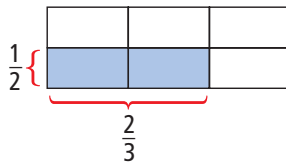
Determine $\frac{2}{3} \times \frac{1}{2}$.

Solution

Draw a rectangle. Draw line segments to cut its length into thirds.



Draw a line segment to cut the width of the rectangle into halves.



The diagram is like the result of paper folding.

Literacy Link

Understanding Common Denominators

For $\frac{1}{2}$ and $\frac{2}{3}$, a common denominator is 6, which is a common multiple of 2 and 3.

$$\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$$

Write $\frac{2}{6}$ in lowest terms.

$$\frac{2}{6} = \frac{1}{3}$$

$$\text{So, } \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

The product of two proper fractions is less than either of the fractions. You can check this relationship using common denominators.

$$\frac{2}{3} = \frac{4}{6} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{2}{6} < \frac{4}{6} \quad \frac{2}{6} < \frac{3}{6}$$

Show You Know

Determine each product using diagrams.

a) $\frac{1}{2} \times \frac{1}{2}$ b) $\frac{1}{3} \times \frac{3}{4}$

Example 3: Multiply Using a Rule

Estimate and calculate $\frac{8}{15} \times \frac{5}{6}$.

Solution

Decide whether each fraction is closer to 0, $\frac{1}{2}$, or 1.



$$\frac{8}{15} \approx \frac{1}{2} \quad \frac{5}{6} \approx 1$$

Then estimate the product.

$$\begin{aligned} \frac{8}{15} \times \frac{5}{6} &\approx \frac{1}{2} \times 1 \\ &\approx \frac{1}{2} \end{aligned}$$

To multiply fractions, multiply the numerators and multiply the denominators.

$$\begin{aligned} \frac{8}{15} \times \frac{5}{6} &= \frac{40}{90} \\ &= \frac{4}{9} \end{aligned}$$

The answer seems reasonable, because it is close to the estimate of $\frac{1}{2}$.

Show You Know

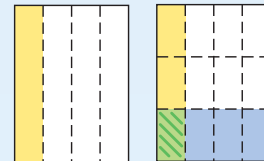
Estimate and calculate.

a) $\frac{3}{5} \times \frac{2}{9}$ b) $\frac{5}{6} \times \frac{4}{5}$

Key Ideas

- You can multiply two proper fractions using paper folding.

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$



- You can multiply two proper fractions using diagrams.

$$\frac{1}{2} \left\{ \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \color{green}{\square} & \color{green}{\square} & \color{green}{\square} & \square \\ \hline \end{array} \right. \quad \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$\underbrace{\hspace{10em}}_{\frac{3}{4}}$

- A rule for multiplying two proper fractions is to multiply the numerators and multiply the denominators.

$$\frac{3}{5} \times \frac{5}{6} = \frac{15}{30} \text{ or } \frac{1}{2}$$

- You can estimate the product of two proper fractions by first deciding whether each fraction is closer to 0, $\frac{1}{2}$, or 1.

$$\frac{3}{7} \approx \frac{1}{2} \quad \frac{4}{7} \approx \frac{1}{2} \quad \frac{3}{7} \times \frac{4}{7} \approx \frac{1}{4}$$

Communicate the Ideas

- Model $\frac{2}{3} \times \frac{1}{3}$ using manipulatives or diagrams.
 - Which method did you choose? Explain why you chose it.
- Brendan calculated $\frac{3}{5} \times \frac{2}{5}$ as follows:
$$\frac{3}{5} \times \frac{2}{5} = \frac{6}{5}$$
 - What mistake did he make?
 - How could you use estimation to show Brendan that he made a mistake?
 - What is the correct product?

Check Your Understanding

Practise

For help with #3 and #4, refer to Examples 1 and 2 on pages 211–212.

- Determine each product using paper folding or diagrams.
 - $\frac{5}{6} \times \frac{1}{2}$
 - $\frac{3}{4} \times \frac{5}{6}$
- Use paper folding or diagrams to determine each product.
 - $\frac{1}{4} \times \frac{2}{3}$
 - $\frac{7}{10} \times \frac{1}{2}$

For help with #5 and #6, refer to Example 3 on page 213.

- Estimate and calculate each product. Express your answer in lowest terms.
 - $\frac{3}{8} \times \frac{2}{3}$
 - $\frac{3}{7} \times \frac{1}{6}$
 - $\frac{3}{4} \times \frac{3}{4}$
- Estimate and calculate each product. Express your answer in lowest terms.
 - $\frac{2}{5} \times \frac{4}{5}$
 - $\frac{7}{8} \times \frac{4}{5}$
 - $\frac{3}{4} \times \frac{4}{9}$

Apply

- Tamar had $\frac{1}{2}$ of an apple pie in her refrigerator. She ate $\frac{1}{4}$ of this piece of pie. What fraction of a whole pie did she eat?
- Marius spends $\frac{1}{3}$ of his time sleeping. While he is asleep, he dreams for $\frac{1}{4}$ of the time.
 - For what fraction of his time is Marius dreaming?
 - For how many hours a day is Marius dreaming?
- About $\frac{1}{20}$ of the people in the world live in Canada or the United States. Of the people who live in Canada or the United States, about $\frac{1}{10}$ live in Canada. What fraction of the people in the world live in Canada?

10. At the age of four, the average person is about $\frac{3}{5}$ as tall as they will be as an adult. At birth, the average person is about $\frac{1}{2}$ as tall as they will be at the age of four. For the average person, what fraction is their height at birth of their height as an adult?

11. When the Summer Olympic and Paralympic Games were held in Athens, Greece, paralympic athletes won $\frac{6}{7}$ of Canada's total medals. Of the medals that Canadian paralympic athletes won, $\frac{7}{18}$ were gold medals.

a) What fraction of Canada's total medals were gold medals won by paralympic athletes?

b) Canada won a total of 84 medals. How many gold medals did Canadian paralympic athletes win?



12. Write a word problem that you can solve using the expression $\frac{3}{4} \times \frac{1}{2}$.

Extend

13. For a standard deck of 52 playing cards, the probability of randomly drawing a red card is $\frac{1}{2}$. The probability of randomly drawing a face card (jack, queen, or king) is $\frac{12}{52}$. What is the probability of randomly drawing a face card that is red?

14. Calculate. Express the product in lowest terms.

a) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

b) $\frac{2}{3} \times \frac{1}{4} \times \frac{2}{5}$

c) $\frac{3}{4} \times \frac{3}{4} \times \frac{2}{9}$

d) $\frac{5}{6} \times \frac{3}{8} \times \frac{7}{10}$

15. Copy each equation. Complete it using a fraction in lowest terms.

a) $\frac{\blacksquare}{\blacksquare} \times \frac{1}{2} = \frac{5}{16}$

b) $\frac{\blacksquare}{\blacksquare} \times \frac{3}{7} = \frac{1}{3}$

c) $\frac{2}{3} \times \frac{\blacksquare}{\blacksquare} = \frac{1}{2}$

d) $\frac{3}{4} \times \frac{\blacksquare}{\blacksquare} = \frac{5}{8}$

16. Use the sum and the product of two fractions to identify the fractions.

a) sum $\frac{1}{2}$; product $\frac{1}{16}$

b) sum $\frac{5}{6}$; product $\frac{1}{6}$

c) sum $\frac{2}{3}$; product $\frac{1}{12}$

MATH LINK

The area of British Columbia is about $\frac{1}{10}$ of the area of Canada. The Pacific Maritime ecozone covers about $\frac{1}{5}$ of the area of British Columbia. What fraction of the area of Canada does the Pacific Maritime ecozone cover?



6.4

Multiplying Improper Fractions and Mixed Numbers

Focus on...

After this lesson, you will be able to...

- multiply two improper fractions or mixed numbers
- solve problems involving the multiplication of improper fractions or mixed numbers



WWW Web Link

To find out more about provincial and territorial flags, go to www.mathlinks8.ca and follow the links. For which of the provincial and territorial flags does the width equal half the length?

The length of the flag of British Columbia is $1\frac{2}{3}$ times its width. How would you determine the length of a flag that is 90 cm wide?

Explore the Math

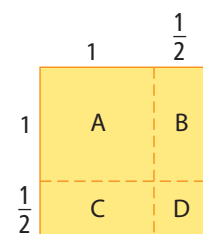
Did You Know?

The study of flags is called vexillology, after the Latin word for flag, *vexillum*.

How can you multiply two improper fractions or mixed numbers?

1. The diagram shows a way to model the multiplication $1\frac{1}{2} \times 1\frac{1}{2}$.

- a) What is the total area of the large square? Show your method. Express the total area as a mixed number in lowest terms.
- b) Write an equation to represent the multiplication.



2. Model each of the following multiplications using a similar model to the one used in #1. Express each total as a mixed number in lowest terms.

- a) $2\frac{1}{2} \times 2\frac{1}{2}$
- b) $1\frac{1}{4} \times 1\frac{1}{4}$
- c) $1\frac{1}{2} \times 1\frac{1}{4}$

Literacy Link

Mixed Numbers in Lowest Terms

A mixed number is in lowest terms when the fraction is in lowest terms.

For example, $3\frac{4}{8}$ expressed in lowest terms is $3\frac{1}{2}$.

$$\begin{array}{c} \div 4 \\ \curvearrowright \\ \frac{4}{8} = \frac{1}{2} \\ \curvearrowleft \\ \div 4 \end{array}$$

3. a) Copy the table. Use your results from #1 and #2 to complete it. The first line is partially completed for you.

Multiplication of Mixed Numbers	Product Expressed as a Mixed Number	Multiplication of Improper Fractions	Product Expressed as an Improper Fraction
$1\frac{1}{2} \times 1\frac{1}{2}$		$\frac{3}{2} \times \frac{3}{2}$	
$2\frac{1}{2} \times 2\frac{1}{2}$			
$1\frac{1}{4} \times 1\frac{1}{4}$			
$1\frac{1}{2} \times 1\frac{1}{4}$			

- b) Write a rule to multiply two improper fractions?
 c) How is your rule the same or different from the rule you developed for multiplying two proper fractions? Explain.

Literacy Link

Converting Improper Fractions and Mixed Numbers

Convert by using the denominator to decide the number of parts in one whole.

In $\frac{11}{4}$, one whole is $\frac{4}{4}$.

$$\frac{11}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$

$$\frac{11}{4} = 2\frac{3}{4}$$

In $3\frac{2}{5}$, one whole is $\frac{5}{5}$.

$$3\frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$$

$$3\frac{2}{5} = \frac{17}{5}$$

Reflect on Your Findings

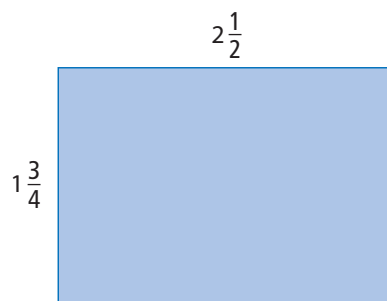
4. How can you multiply two improper fractions or mixed numbers by using a model or a rule?

Example 1: Multiply Mixed Numbers Using a Model

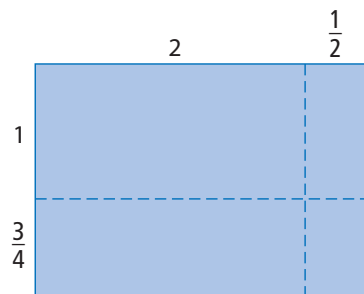
Determine $2\frac{1}{2} \times 1\frac{3}{4}$.

Solution

Draw a rectangle.



Draw a line segment to separate each dimension into a whole number and a proper fraction.



Strategies

Model It

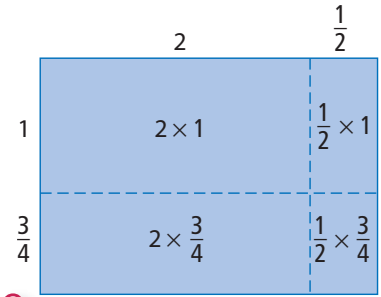
Literacy Link

A whole number can be written as a fraction with a denominator of 1.

For example, $2 = \frac{2}{1}$

Show the area of each of the four regions in the diagram.

$$\begin{aligned} 2 \times \frac{3}{4} &= \frac{2}{1} \times \frac{3}{4} \\ &= \frac{6}{4} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2} \end{aligned}$$



Calculate the area of each region and add the areas.

$$2 \times 1 = 2 \quad 2 \times \frac{3}{4} = 1\frac{1}{2} \quad \frac{1}{2} \times 1 = \frac{1}{2} \quad \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$$2 + 1\frac{1}{2} + \frac{1}{2} + \frac{3}{8} = 4\frac{3}{8}$$

$$\text{So, } 2\frac{1}{2} \times 1\frac{3}{4} = 4\frac{3}{8}$$

Show You Know

Determine each product using a model.

a) $1\frac{3}{4} \times 1\frac{3}{4}$ b) $2\frac{1}{4} \times 2\frac{1}{4}$ c) $1\frac{1}{4} \times 1\frac{1}{3}$

Example 2: Multiply Mixed Numbers Using a Rule

Estimate and calculate $4\frac{1}{2} \times 2\frac{1}{3}$.

Solution

Estimate the product by multiplying the whole numbers closest to each mixed number.

$$\begin{aligned} 4\frac{1}{2} \times 2\frac{1}{3} &\approx 5 \times 2 \\ &\approx 10 \end{aligned}$$

To calculate $4\frac{1}{2} \times 2\frac{1}{3}$, write the mixed numbers as improper fractions.

$$4\frac{1}{2} \times 2\frac{1}{3} = \frac{9}{2} \times \frac{7}{3}$$

$$\begin{aligned} 4\frac{1}{2} &= \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} \\ &= \frac{9}{2} \end{aligned} \quad \begin{aligned} 2\frac{1}{3} &= \frac{3}{3} + \frac{3}{3} + \frac{1}{3} \\ &= \frac{7}{3} \end{aligned}$$

To multiply the fractions, multiply the numerators and multiply the denominators.

$$\begin{aligned} \frac{9}{2} \times \frac{7}{3} &= \frac{63}{6} \\ &= \frac{21}{2} \\ &= 10\frac{1}{2} \end{aligned}$$

Write this fraction in lowest terms.

The answer is close to the estimate.

The product of two mixed numbers or improper fractions is greater than either of them.

$$10\frac{1}{2} > 4\frac{1}{2} \quad 10\frac{1}{2} > 2\frac{1}{3}$$

Show You Know

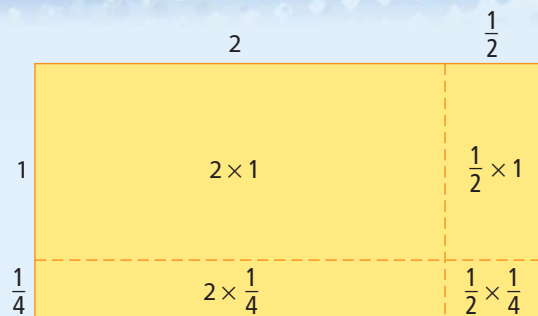
Estimate and calculate.

a) $1\frac{1}{10} \times 3\frac{1}{2}$ b) $1\frac{1}{4} \times 3\frac{2}{3}$

Key Ideas

- You can model the multiplication of two mixed numbers or improper fractions using partial areas of a rectangle.

$$\begin{aligned}2\frac{1}{2} \times 1\frac{1}{4} &= 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8} \\ &= 3\frac{1}{8}\end{aligned}$$



- You can estimate the product of two mixed numbers or improper fractions by multiplying the whole numbers closest to them.

$$\begin{aligned}3\frac{1}{4} \times 1\frac{3}{4} &\approx 3 \times 2 \\ &\approx 6\end{aligned}$$

- A rule for multiplying two mixed numbers is to express them as improper fractions and then multiply the numerators and multiply the denominators.

$$\begin{aligned}1\frac{2}{3} \times 2\frac{1}{5} &= \frac{5}{3} \times \frac{11}{5} \\ &= \frac{55}{15} \text{ or } \frac{11}{3}\end{aligned}$$

Communicate the Ideas

- Henri multiplied $2\frac{1}{2} \times 3\frac{1}{4}$ as follows: $2 \times 3 = 6$ and $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$, so $2\frac{1}{2} \times 3\frac{1}{4} = 6\frac{1}{8}$.
 - What mistake did Henri make?
 - What is the correct product?
- To express $4\frac{2}{3}$ as an improper fraction, Naomi determined the numerator by calculating $3 \times 4 + 2$.
 - Explain why Naomi's method works.
 - Use your explanation to write a rule for expressing a mixed number as an improper fraction. Test your rule.
 - Write a rule for expressing an improper fraction as a mixed number. Test your rule.

3. Moira multiplied $2\frac{1}{3} \times 2\frac{1}{2}$ as follows:

$$\begin{aligned}2\frac{1}{3} \times 2\frac{1}{2} &= \frac{7}{3} \times \frac{5}{2} \\ &= \frac{14}{6} \times \frac{15}{6} \\ &= \frac{210}{36} \\ &= \frac{35}{6} \\ &= 5\frac{5}{6}\end{aligned}$$

- Was her final answer correct?
- How did she make the calculation longer than necessary

Check Your Understanding

Practise

- Express each improper fraction as a mixed number.
a) $\frac{11}{3}$ b) $\frac{17}{6}$ c) $\frac{25}{2}$ d) $\frac{8}{5}$
- Express each mixed number as an improper fraction.
a) $4\frac{3}{4}$ b) $2\frac{7}{8}$ c) $6\frac{1}{3}$ d) $3\frac{4}{7}$

For help with #6 and #7, refer to Example 1 on pages 217–218.

- Use a model to determine each product.
a) $1\frac{1}{3} \times \frac{3}{4}$ b) $2\frac{1}{2} \times 1\frac{3}{5}$
c) $1\frac{1}{3} \times 1\frac{1}{2}$ d) $2\frac{1}{2} \times 2\frac{1}{4}$
- Determine each product using a model.
a) $\frac{1}{2} \times 2\frac{1}{2}$ b) $2\frac{1}{3} \times 2\frac{1}{3}$
c) $1\frac{1}{2} \times 2\frac{1}{3}$ d) $1\frac{1}{5} \times 1\frac{1}{2}$

For help with #8 and #9, refer to Example 2 on page 218.

- Estimate and calculate.
a) $\frac{4}{5} \times \frac{10}{7}$ b) $5 \times 3\frac{3}{4}$ c) $2\frac{1}{5} \times 1\frac{2}{3}$
- Estimate and calculate.
a) $\frac{8}{3} \times \frac{11}{6}$ b) $2\frac{5}{6} \times 4$ c) $6\frac{1}{2} \times 3\frac{1}{2}$

Apply

- Two and a half laps of a running track equal 1 km. How many laps equal 3 km?
- Earth turns on its axis once every 24 h. How many hours does Earth take to complete $2\frac{1}{4}$ turns?
- On a day in Winnipeg with $10\frac{1}{2}$ h of daylight, it was sunny for $\frac{1}{3}$ of that time. For how many hours was it sunny that day?

13. Alexa takes $\frac{1}{4}$ h to ride her bicycle to her friend's house. If Alexa walks instead, the trip takes her $2\frac{1}{2}$ times as long. How long does Alexa take to walk to her friend's house
- in hours?
 - in minutes?



14. In Eric's apartment, the living room is $1\frac{3}{4}$ times as long and $2\frac{1}{2}$ times as wide as the den. Eric is buying the same type of carpet for both rooms. How many times as much will the carpet cost for the living room as for the den?
15. Andreas has \$18. Bonnie has $1\frac{2}{3}$ times as much as Andreas. Cheryl has $1\frac{3}{5}$ times as much as Bonnie. How much money do they have altogether?

16. A corner store buys goods at the wholesale price and sells them for $\frac{7}{5}$ of the wholesale price. The wholesale price of a case of 12 cans of stew is \$15. For how much does the store sell one can of stew?
17. If you multiply a mixed number and a proper fraction, how does each value compare with the value of the product?
18. Create your own word problem that involves the multiplication of two mixed numbers. Make sure that you can solve your problem. Give your problem to a classmate to solve.

Extend

19. Describe each pattern. Then write the next three terms in each pattern.
- $4\frac{1}{3}, 2\frac{1}{6}, 1\frac{1}{12}, \frac{13}{24}, \dots$
 - $4, 6, 9, 13\frac{1}{2}, \dots$
20. Calculate.
- $4 \times 1\frac{1}{2} \times 2\frac{1}{2}$
 - $\frac{2}{3} \times 3\frac{1}{3} \times 4\frac{1}{2}$
 - $2\frac{3}{4} \times 1\frac{1}{3} \times 3\frac{1}{2}$
 - $1\frac{1}{6} \times 1\frac{2}{5} \times 2\frac{2}{7}$
21. Copy each equation. Use a mixed number to complete it.
- $1\frac{2}{3} \times \blacksquare = 2\frac{1}{2}$
 - $\blacksquare \times 2\frac{1}{6} = 2\frac{3}{5}$
 - $\blacksquare \times 1\frac{1}{4} = 3\frac{1}{8}$
 - $2\frac{1}{3} \times \blacksquare = 5\frac{5}{6}$

MATH LINK

The Hudson Plains ecozone contains most of Canada's wetlands. This ecozone covers about $\frac{1}{26}$ of the area of Canada. The Northern Arctic ecozone is one of the world's largest Arctic ecosystems. This ecozone is about $3\frac{9}{10}$ times as big as the Hudson Plains ecozone. What fraction of the area of Canada does the Northern Arctic ecozone cover?



6.5

Dividing Fractions and Mixed Numbers

Focus on...

After this lesson, you will be able to...

- divide two fractions or mixed numbers
- solve problems involving the division of fractions or mixed numbers



Russia and Canada are the two countries with the largest areas in the world. How are the fraction and the mixed number in the following two statements related?

The area of Canada is about $\frac{3}{5}$ of the area of Russia.

The area of Russia is about $1\frac{2}{3}$ times the area of Canada.

Explore the Math

How can you divide a fraction by a fraction?

Work with a partner.

1. When you divide 8 by 4, you determine how many 4s there are in 8.
 $8 \div 4 = 2$.
 - a) Show how you could determine how many $\frac{1}{2}$ s there are in 3.
 - b) Copy and complete the equation $3 \div \frac{1}{2} = \blacksquare$.

2. a) Show how you could determine how many $\frac{1}{4}$ s there are in 2.
 b) Copy and complete the equation $2 \div \frac{1}{4} = \blacksquare$.

3. a) Show how you could determine how many $\frac{1}{4}$ s there are in $\frac{3}{4}$.
 b) Copy and complete the equation $\frac{3}{4} \div \frac{1}{4} = \blacksquare$.

4. a) Show how you could determine how many $\frac{1}{6}$ s are in $\frac{2}{3}$.

b) Copy and complete the equation $\frac{2}{3} \div \frac{1}{6} = \blacksquare$.

5. a) Copy the table. Complete the divisions in the first column.

Division	Division With Equal Denominators
$\frac{3}{4} \div \frac{1}{4} = \blacksquare$	$\frac{3}{4} \div \frac{1}{4} = \blacksquare$
$\frac{2}{3} \div \frac{1}{3} = \blacksquare$	$\frac{2}{3} \div \frac{1}{3} = \blacksquare$
$\frac{8}{9} \div \frac{2}{9} = \blacksquare$	$\frac{8}{9} \div \frac{2}{9} = \blacksquare$
$\frac{2}{3} \div \frac{1}{6} = \blacksquare$	$\frac{4}{6} \div \frac{1}{6} = \blacksquare$
$\frac{1}{2} \div \frac{1}{12} = \blacksquare$	$\frac{6}{12} \div \frac{1}{12} = \blacksquare$
$\frac{3}{4} \div \frac{3}{8} = \blacksquare$	$\frac{6}{8} \div \frac{3}{8} = \blacksquare$

b) How are the divisions in the second column related to the divisions in the first column? Explain.

c) Complete the second column.

d) Write a rule for dividing two fractions using common denominators.

6. a) Copy the table. Complete the divisions in the first column.

Division	Multiplication
$\frac{3}{4} \div \frac{1}{4} = \blacksquare$	$\frac{3}{4} \times \frac{4}{1} = \blacksquare$
$\frac{2}{3} \div \frac{1}{3} = \blacksquare$	$\frac{2}{3} \times \frac{3}{1} = \blacksquare$
$\frac{8}{9} \div \frac{2}{9} = \blacksquare$	$\frac{8}{9} \times \frac{9}{2} = \blacksquare$
$\frac{2}{3} \div \frac{1}{6} = \blacksquare$	$\frac{2}{3} \times \frac{6}{1} = \blacksquare$
$\frac{1}{2} \div \frac{1}{12} = \blacksquare$	$\frac{1}{2} \times \frac{12}{1} = \blacksquare$
$\frac{3}{4} \div \frac{3}{8} = \blacksquare$	$\frac{3}{4} \times \frac{8}{3} = \blacksquare$

b) Complete the multiplications in the second column.

c) How are the multiplications in the second column related to the divisions in the first column? Explain.

d) Write a rule for dividing by a fraction using multiplication.



Reflect on Your Findings

7. Which method do you prefer to use to divide a fraction by a fraction? Why?

Example 1: Divide Using Diagrams

Determine $\frac{2}{3} \div \frac{1}{4}$.

Solution

Use diagrams to determine how many $\frac{1}{4}$ s are in $\frac{2}{3}$.

The diagram shows that the number of $\frac{1}{4}$ s in $\frac{2}{3}$ is between 2 and 3.

A common denominator for $\frac{1}{4}$ and $\frac{2}{3}$ is 12. So use a rectangle divided into twelfths.

$$\frac{2}{3} \div \frac{1}{4} = 2\frac{2}{3} \text{ or } \frac{8}{3}$$



In $\frac{8}{12}$, there are 2 whole groups of $\frac{3}{12}$, plus $\frac{2}{3}$ of another group.

When you divide a number by a proper fraction, the quotient is greater than the original number.

$$\frac{8}{3} > \frac{2}{3}$$

Show You Know

Determine using diagrams.

a) $\frac{3}{4} \div \frac{1}{3}$ b) $1\frac{1}{4} \div \frac{3}{8}$ c) $\frac{1}{10} \div \frac{1}{5}$

Example 2: Divide Using a Rule

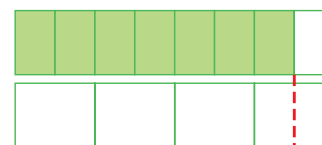
Estimate and calculate.

a) $\frac{7}{8} \div \frac{1}{4}$ b) $2\frac{1}{2} \div 3\frac{3}{4}$

Solution



a) The diagram shows that the number of $\frac{1}{4}$ s in $\frac{7}{8}$ is between 3 and 4.



You can calculate the quotient using either of the following methods.

Method 1: Divide Using a Common Denominator

To divide fractions, write them with a common denominator and divide the numerators.

$$\begin{aligned} \frac{7}{8} \div \frac{1}{4} &= \frac{7}{8} \div \frac{2}{8} \\ &= \frac{7}{2} \text{ or } 3\frac{1}{2} \end{aligned}$$

$\frac{1}{4} = \frac{2}{8}$

Method 2: Divide Using Multiplication

To divide by a fraction, multiply by its reciprocal.

$$\begin{aligned} \frac{7}{8} \div \frac{1}{4} &= \frac{7}{8} \times \frac{4}{1} \\ &= \frac{28}{8} \\ &= \frac{7}{2} \text{ or } 3\frac{1}{2} \end{aligned}$$

reciprocal

- the multiplier of a number to give a product of 1
- $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, because $\frac{3}{2} \times \frac{2}{3} = 1$
- the result of switching the numerator and denominator in a fraction

b) You can estimate the quotient by dividing the whole numbers closest to the mixed numbers.



$$2\frac{1}{2} \div 3\frac{3}{4} \approx 3 \div 4$$

$$\approx \frac{3}{4}$$

You can calculate the quotient using either of the following methods.

Method 1: Divide Using a Common Denominator

$$2\frac{1}{2} \div 3\frac{3}{4} = \frac{5}{2} \div \frac{15}{4}$$

$$= \frac{10}{4} \div \frac{15}{4}$$

$$= \frac{10}{15}$$

$$= \frac{2}{3}$$

Method 2: Divide Using Multiplication

$$2\frac{1}{2} \div 3\frac{3}{4} = \frac{5}{2} \div \frac{15}{4}$$

$$= \frac{5}{2} \times \frac{4}{15}$$

$$= \frac{20}{30}$$

$$= \frac{2}{3}$$

$\frac{4}{15}$ is the reciprocal of $\frac{15}{4}$.

When you divide a number by an improper fraction or mixed number, the quotient is less than the original number.

$$\frac{2}{3} < 2\frac{1}{2}$$

Show You Know

Estimate and calculate.

a) $\frac{4}{5} \div \frac{3}{10}$ b) $\frac{2}{9} \div \frac{5}{6}$ c) $3\frac{1}{6} \div 1\frac{2}{3}$

Example 3: Apply Division With Fractions

The baby teeth, or milk teeth, that develop in childhood are replaced by larger teeth as people mature. A full set of teeth for a child has $\frac{5}{8}$ as many teeth as a full set of teeth for an adult. There are 20 teeth in a full set for a child. How many teeth are there in a full set for an adult?

Solution

Divide 20 by $\frac{5}{8}$ to determine the number of adult teeth.

$$20 \div \frac{5}{8} = \frac{20}{1} \div \frac{5}{8}$$

$$= \frac{20}{1} \times \frac{8}{5}$$

$$= \frac{160}{5}$$

$$= 32$$

Instead of multiplying, you could divide using common denominators.

$$20 \div \frac{5}{8} = \frac{20}{1} \div \frac{5}{8}$$

$$= \frac{160}{8} \div \frac{5}{8}$$

There are 32 teeth in a full set for an adult.

Check:

Use multiplication to check the division.

$$\frac{5}{8} \times 32 = \frac{160}{8}$$

$$= 20$$

Did You Know?

A beaver, like other rodents, has only one set of teeth in its lifetime. A beaver's front teeth can grow by over a metre a year. The gnawing that the beaver does to cut trees and to eat wears down its teeth and keeps them the right length.

20 is $\frac{5}{8}$ of the number of adult teeth, so there are more than 20 adult teeth.

Dividing 20 by $\frac{5}{8}$ will result in a quotient greater than 20.

$$20 \div \frac{5}{8} \approx 20 \div \frac{1}{2}$$

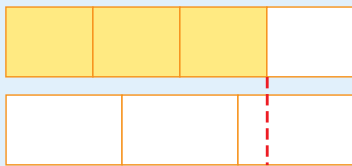
$$\approx 40$$

Show You Know

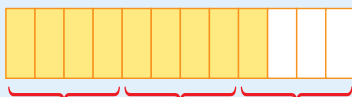
If one serving is $\frac{1}{6}$ of a tray of lasagna, how many servings are in three trays of lasagna?

Key Ideas

- You can estimate and determine the quotient of two fractions using diagrams.



$\frac{3}{4} \div \frac{1}{3}$ is between 2 and 3.



$$\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4} \text{ or } \frac{9}{4}$$

- You can estimate the quotient of two improper fractions or mixed numbers by dividing the whole numbers closest to them.

$$\begin{aligned} 5\frac{1}{4} \div 3\frac{1}{2} &\approx 5 \div 4 \\ &\approx \frac{5}{4} \text{ or } 1\frac{1}{4} \end{aligned}$$

- To divide two fractions, you can write them with a common denominator and divide the numerators.

$$\begin{aligned} \frac{7}{10} \div \frac{3}{5} &= \frac{7}{10} \div \frac{6}{10} & 1\frac{3}{4} \div 2\frac{1}{2} &= \frac{7}{4} \div \frac{5}{2} \\ &= \frac{7}{6} \text{ or } 1\frac{1}{6} & &= \frac{7}{4} \div \frac{10}{4} \\ & & &= \frac{7}{10} \end{aligned}$$

- To divide by a fraction, you can multiply by its reciprocal.

$$\begin{aligned} \frac{7}{10} \div \frac{3}{5} &= \frac{7}{10} \times \frac{5}{3} & 1\frac{3}{4} \div 2\frac{1}{2} &= \frac{7}{4} \div \frac{5}{2} \\ &= \frac{35}{30} & &= \frac{7}{4} \times \frac{2}{5} \\ &= \frac{7}{6} \text{ or } 1\frac{1}{6} & &= \frac{14}{20} \\ & & &= \frac{7}{10} \end{aligned}$$

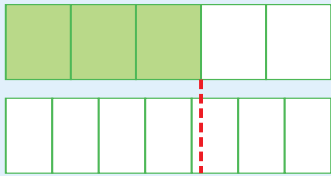
Communicate the Ideas

- Anna divided $\frac{5}{6}$ by $\frac{1}{2}$ and stated that the quotient is $\frac{5}{12}$.
 - What was Anna's mistake?
 - What is the correct quotient? Use a diagram to show how you know.
- Mike carried out the division $\frac{3}{4} \div \frac{2}{3}$ as follows.

$$\begin{aligned}\frac{3}{4} \div \frac{2}{3} &= \frac{4}{3} \times \frac{2}{3} \\ &= \frac{8}{9}\end{aligned}$$

Do you agree with Mike's method and answer? Explain.

- Explain how the diagram shows that the quotient of $\frac{3}{5} \div \frac{1}{7}$ is between 4 and 5.



- Does $2\frac{1}{2} \div 1\frac{1}{2}$ equal $1\frac{1}{2} \div 2\frac{1}{2}$?
 - If the quotients in part a) are not equal, how are they related?

Check Your Understanding

Practise

For help with #5 and #6, refer to Example 1 on page 224.

- Determine each quotient using diagrams.

$$\begin{array}{ll} \text{a) } \frac{5}{8} \div \frac{1}{4} & \text{b) } \frac{1}{4} \div \frac{1}{3} \\ \text{c) } 1\frac{1}{2} \div \frac{2}{3} & \text{d) } 2\frac{1}{3} \div \frac{5}{6} \end{array}$$

- Use diagrams to determine each quotient.

$$\begin{array}{ll} \text{a) } \frac{9}{10} \div \frac{1}{5} & \text{b) } \frac{1}{4} \div \frac{3}{8} \\ \text{c) } 1\frac{2}{3} \div \frac{1}{2} & \text{d) } 2\frac{3}{4} \div \frac{2}{3} \end{array}$$

For help with #7 to #10, refer to Example 2 on pages 224–225.

- Divide using a common denominator.
 - $\frac{3}{5} \div \frac{9}{10}$
 - $1\frac{1}{2} \div \frac{5}{6}$
 - $3\frac{1}{3} \div 1\frac{5}{6}$
- Divide using multiplication.
 - $\frac{5}{12} \div \frac{3}{4}$
 - $4\frac{1}{2} \div 1\frac{1}{4}$
 - $10 \div 2\frac{1}{2}$
- Divide.
 - $\frac{3}{4} \div \frac{4}{5}$
 - $1\frac{2}{3} \div 2\frac{5}{6}$
 - $12 \div \frac{3}{4}$

10. Divide.

a) $1\frac{1}{12} \div 2\frac{1}{2}$ b) $\frac{8}{11} \div \frac{4}{5}$ c) $1\frac{3}{8} \div 2\frac{3}{4}$

Apply

For help with #11 to #13, refer to Example 3 on page 225.

11. In a comedy review, each performer has a $\frac{1}{4}$ -h slot. How many performers are there in a 2-h show?
12. It takes $2\frac{1}{2}$ scoops of flour to make one cake. How many cakes do 15 scoops of flour make?
13. Three quarters of a can of apple juice fills six glasses. How many glasses will a whole can of apple juice fill?
14. An incandescent light bulb uses about $4\frac{1}{2}$ times as much energy as a compact fluorescent light bulb to produce the same amount of light. What fraction of the energy used by the incandescent bulb does the fluorescent light bulb use?



15. Shana and Zack painted their rooms using paint in cans of the same size. Shana used $1\frac{1}{2}$ cans of paint. Zack used $2\frac{3}{4}$ cans of paint. How many times as much paint did Zack use as Shana?

16. Of all the land on Earth, about $\frac{3}{10}$ is in Asia and about $\frac{3}{25}$ is in South America. How many times as big as South America is Asia?

17. The average wind speed in Calgary is $\frac{4}{5}$ of the average wind speed in Regina. The average wind speed in Calgary is 16 km/h. What is the average wind speed in Regina?

18. Use examples to explain your answer to each of the following.

- a) Can the reciprocal of a proper fraction be a proper fraction?
- b) Can the product of two proper fractions be greater than 1?
- c) Can the quotient of two proper fractions be greater than 1?

19. a) The world's longest river is the Nile in Africa, with a length of 6825 km. This is about $1\frac{5}{8}$ times as long as the Mackenzie River, which is Canada's longest river.

- a) How long is the Mackenzie River?
- b) The Mackenzie River is about $2\frac{1}{10}$ times as long as the Columbia River. How long is the Columbia River?



20. Russia covers about $\frac{1}{30}$ of the Earth's surface. The area of Russia is about $1\frac{2}{3}$ times the area of Canada. What fraction of the Earth's surface does Canada cover?

21. Suppose a friend knows how to divide by whole numbers, but not by fractions.

a) How could you use the following pattern to show your friend how to calculate $4 \div \frac{1}{2}$?

$$4 \div 8 = \frac{1}{2}$$

$$4 \div 4 = 1$$

$$4 \div 2 = 2$$

$$4 \div 1 = 4$$

$$4 \div \frac{1}{2} = \blacksquare$$

b) Make up a pattern to show your friend how to calculate $9 \div \frac{1}{3}$.

22. Write a word problem that you can solve using the expression $3\frac{3}{4} \div 2\frac{1}{4}$.

Extend

23. It took Svend $9\frac{3}{4}$ min to ski up a slope on a cross-country ski trail and only $2\frac{1}{4}$ min to ski back down the same slope. How many times as fast did he ski down the slope as he skied up it?



24. The three largest islands in Canada are all north of the Arctic Circle. Baffin Island has about $2\frac{1}{3}$ times the area of Victoria Island. Baffin Island has about $2\frac{3}{5}$ times the area of Ellesmere Island. What fraction of the area of Victoria Island is the area of Ellesmere Island?

MATH LINK

The Prairies ecozone includes the Manitoba Plain and the grasslands of southwest Saskatchewan and southeast Alberta. The wettest part of this ecozone is the Manitoba Plain, which has an average annual precipitation of about 70 cm. This amount of precipitation is $2\frac{4}{5}$ of the amount in the dry grasslands.

What is the average annual precipitation in these grasslands?

Did You Know?

The Prairies ecozone contains much of Canada's farmland, but it is vulnerable to droughts.



6.6

Applying Fraction Operations

Focus on...

After this lesson, you will be able to...

- decide when to multiply fractions and when to divide fractions in solving problems
- apply the order of operations to solve problems involving fractions

Gold has been valued since ancient times because of its beauty and its short supply. Canada is one of the world's leading gold producers.

About $\frac{17}{20}$ of the world's gold production is used to make jewellery.

About $\frac{1}{40}$ of the world's gold

production is used to make coins. What operation would you use to determine how many times as much gold is used to make jewellery as is used to make coins?



Explore the Math

Did You Know?

A *gold rush* is a sudden movement of many people to an area where gold has been discovered. Canada's biggest gold rush was the Klondike Gold Rush of 1897–1898 in Yukon Territory. Large amounts of gold were discovered there in 1896 by a group led by Keish, who was a member of the Tagish First Nation. He was also known as Skookum Jim Mason.

How can you decide which operations to use when solving problems involving fractions?

Many objects that appear to be made of pure gold are actually made from mixtures of gold and cheaper metals. The purity of the gold is measured using a unit called the karat. This unit represents the fraction $\frac{1}{24}$. The table shows the fraction of gold and the fraction of other metals in gold objects with two different purities.

Purity of Gold (karats)	Fraction of Gold	Fraction of Other Metals
20	$\frac{5}{6}$	$\frac{1}{6}$
14	$\frac{7}{12}$	$\frac{5}{12}$

1. How would you calculate the fraction of gold from the purity of gold? Explain.
2. How would you calculate the fraction of other metals from the fraction of gold? Explain.

3. For a gold object with a purity of 18 karats, what is
- the fraction of gold, in lowest terms?
 - the fraction of other metals, in lowest terms?
4. Use the meaning of a karat to explain why pure gold is described as 24-karat gold.
5. How would you calculate the purity of gold from the fraction of gold? Explain.
6. What is the purity of gold, in karats, if the fraction of gold is
- $\frac{1}{2}$?
 - $\frac{5}{12}$?

WWW Web Link

To find out more about Canada's gold rushes and the life of Keish, go to www.mathlinks8.ca and follow the links.

Reflect on Your Findings

7. How did you decide which operations to use in #1, #2, and #5? Discuss your ideas with your classmates.

Example 1: Use the Order of Operations

Calculate.

a) $2 \div \frac{1}{4} + 3 \times \frac{1}{2}$ b) $\frac{1}{3} \times (9 - 2) - \frac{5}{6}$ c) $2\frac{1}{4} \div \left(1\frac{3}{4} + 1\frac{1}{4}\right)$

Solution

<p>a) $2 \div \frac{1}{4} + 3 \times \frac{1}{2}$</p> <p>$= 8 + 3 \times \frac{1}{2}$</p> <p>$= 8 + \frac{3}{2}$</p> <p>$= \frac{16}{2} + \frac{3}{2}$</p> <p>$= \frac{19}{2}$ or $9\frac{1}{2}$</p>	<p>Divide.</p> <p>Multiply.</p> <p>Add.</p>	<p>b) $\frac{1}{3} \times (9 - 2) - \frac{5}{6}$</p> <p>$= \frac{1}{3} \times 7 - \frac{5}{6}$</p> <p>$= \frac{7}{3} - \frac{5}{6}$</p> <p>$= \frac{14}{6} - \frac{5}{6}$</p> <p>$= \frac{9}{6}$</p> <p>$= \frac{3}{2}$ or $1\frac{1}{2}$</p>	<p>Brackets.</p> <p>Multiply.</p> <p>Subtract.</p>
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<p>c) $2\frac{1}{4} \div \left(1\frac{3}{4} + 1\frac{1}{4}\right)$</p> <p>$= 2\frac{1}{4} \div 3$</p> <p>$= \frac{9}{4} \times \frac{1}{3}$</p> <p>$= \frac{9}{12}$</p> <p>$= \frac{3}{4}$</p>	<p>Brackets.</p> <p>Divide.</p>
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Literacy Link

The *order of operations* for fractions is the same as for whole numbers and decimals.

- Brackets first.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

Show You Know

Calculate.

a) $7 \times \frac{1}{2} - 2 \div \frac{3}{5}$ b) $\frac{3}{2} \div \left(\frac{1}{2} + \frac{1}{4}\right) \div \frac{3}{4}$ c) $2\frac{1}{4} - \frac{1}{2} \times \left(\frac{3}{4} - \frac{1}{8}\right)$

Literacy Link

To earn time-and-a-half means to be paid for $1\frac{1}{2}$ h for each hour of work done.

Example 2: Apply Fraction Operations

Bev earns \$25/h as a machine operator in a sawmill. For time worked above 40 h in a week, she earns time-and-a-half. How much does Bev earn for working 46 h in a week?

Solution

Method 1: Calculate in Stages

Bev's regular rate of pay is \$25/h. In 46 h, Bev works 40 h at her regular rate of pay and 6 h at time-and-a-half.

Amount earned at regular rate:

$$40 \times 25 = 1000$$

Bev works 6 h at time-and-a-half. Multiply to determine the number of hours Bev is paid for.

$$6 \times 1\frac{1}{2} = 9$$

Amount earned at time-and-a-half:

$$9 \times 25 = 225$$

$$\begin{aligned} \text{Total earnings} &= 1000 + 225 \\ &= 1225 \end{aligned}$$

Bev earns \$1225 for working 46 h in a week.

Method 2: Evaluate One Expression

Bev's regular rate of pay is \$25/h. In 46 h, Bev works 40 h at her regular rate of pay and 6 h at time-and-a-half.

For 6 h at time-and-a-half, Bev is paid for $1\frac{1}{2} \times 6$ h.

An expression that represents her total earnings is:

$$25 \times \left(40 + 1\frac{1}{2} \times 6\right)$$

Evaluate the expression using the order of operations.

$$25 \times \left(40 + 1\frac{1}{2} \times 6\right) \quad \text{Brackets.}$$

$$= 25 \times 49 \quad \text{Multiply.}$$

$$= 1225$$

Bev earns \$1225 for working 46 h in a week.

Show You Know

Ron earns \$15/h as a security guard. For time worked above 35 h in a week, he earns time-and-a-third. How much does Ron earn for working 41 h in a week?

Key Ideas

- You need to decide which operation(s) to perform on fractions to solve problems.
- Some fraction problems can involve the order of operations.
- The order of operations for fractions is the same as for whole numbers and decimals.
 - Brackets first.
 - Multiply and divide in order from left to right.
 - Add and subtract in order from left to right.

Communicate the Ideas

1. Ranjeet is entering a competition to win some gold coins. She must answer the following skill-testing question.

What is the value of $10 - 2 \times \frac{1}{2}$?

She is unsure if the correct answer is 4 or 9.

- a) How could Ranjeet determine a possible answer of 4?
 - b) How could Ranjeet determine a possible answer of 9?
 - c) What is the correct answer? Explain.
2. Dave and Manuel were comparing their solutions to the following problem.

Three quarters of a number is 6. What is the number?

Dave evaluated $\frac{3}{4} \times 6$ to get an answer of $4\frac{1}{2}$.

Manuel evaluated $6 \div \frac{3}{4}$ to get an answer of 8.

Which answer is correct? Explain.

3. Mia evaluated the expression $\left(\frac{1}{2} + \frac{1}{4}\right) \times \frac{5}{3}$ to equal $\frac{11}{12}$.
 - a) What mistake did she make?
 - b) What is the correct value?

Check Your Understanding

Practise

For help with #4 and #5, refer to Example 1 on page 231.

4. Calculate.

a) $\frac{3}{4} - \frac{1}{2} \times \frac{2}{3}$ b) $2\frac{1}{5} \div \left(\frac{4}{5} - \frac{1}{4}\right)$

c) $3\frac{1}{2} + 2\frac{1}{2} \times \left(1\frac{1}{4} - \frac{3}{4}\right)$

5. Calculate.

a) $\left(\frac{5}{6} + \frac{2}{3}\right) \times \frac{3}{7}$ b) $\frac{1}{2} + \frac{3}{5} \div \frac{3}{4} \div \frac{2}{5}$

c) $1\frac{2}{5} \times 2\frac{1}{2} \div \left(1\frac{1}{8} - \frac{2}{3}\right)$

Apply

For help with #6, refer to Example 2 on page 232.

6. Leo earns \$16/h as a gardener in a city park. For time worked above 35 h in a week, he earns time-and-a-half. How much does he earn for each of the following numbers of hours worked in a week?

a) 36 h b) 39 h c) 42 h d) $37\frac{1}{2}$ h

7. Two thirds of the land on a farm is used for grazing beef cattle. The rest of the land is used to grow crops. Half of the land for crops is used to grow corn. What fraction of the land on the farm is used to grow corn?

8. Melissa and Shinzo found $\frac{1}{2}$ a pitcher of iced tea in the fridge. They equally shared $\frac{3}{4}$ of the iced tea.

- a) What fraction of a pitcher of iced tea did each of them drink?
 b) What fraction of a pitcher of iced tea was left over?

9. Five sevenths of the 28 students in a grade 8 class visited a science museum on a field trip. How many students did not go on the trip? Solve the problem in two different ways.



10. Brass is an alloy that contains the metals copper and zinc. Copper typically accounts for $\frac{3}{5}$ of the mass of a piece of brass.

- a) What is the mass of copper in 175 g of brass?
 b) What mass of brass contains 90 g of copper?
 c) What mass of brass contains 50 g of zinc?

11. The advertising space in a hockey team's yearbook is sold in fractions of a page. The advertising space sold in one edition of the yearbook is shown in the table.

Size of Advertisement	Price	Number Sold
$\frac{1}{2}$ page	\$110	3
$\frac{1}{4}$ page	\$60	5
$\frac{1}{8}$ page	\$35	12

Calculate the following.

- a) the total number of pages of advertising sold
 b) the total revenue from advertising
 c) the average revenue per page of advertising sold

12. One week, Marjorie spent $\frac{1}{2}$ of her allowance on a music video, $\frac{1}{4}$ of her allowance on a T-shirt, and $\frac{1}{8}$ of her allowance on bus fares. She had \$5 of her allowance left at the end of the week. How much was her allowance that week?

13. Add one pair of brackets to the left side of each equation to make the equation true.

a) $\frac{5}{2} \times \frac{3}{5} - \frac{2}{5} + \frac{1}{2} = 1$

b) $1\frac{1}{2} + 2\frac{1}{2} \div \frac{3}{4} - \frac{1}{8} = 5\frac{1}{2}$

c) $\frac{2}{3} - \frac{1}{6} + \frac{5}{6} \div \frac{16}{9} = \frac{3}{4}$

14. Here is a way of using four $\frac{1}{2}$ s and the order of operations to write an expression that equals 2.

$$\frac{1}{2} \div \frac{1}{2} + \frac{1}{2} \div \frac{1}{2}$$

Use four $\frac{1}{2}$ s and the order of operations to write expressions with each of the following values. Compare your expressions with your classmates' expressions.

- | | | |
|------------------|------------------|-------------------|
| a) 0 | b) 1 | c) $\frac{1}{4}$ |
| d) 3 | e) $\frac{1}{2}$ | f) 4 |
| g) $\frac{5}{8}$ | h) $\frac{5}{4}$ | i) $2\frac{1}{2}$ |

Extend

15. The mean of four fractions is $\frac{2}{3}$. Three of the fractions are $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$. What is the fourth fraction?

Literacy Link

The mean of a set of fractions is their sum divided by the number of fractions.

The mean of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8}$ is $\left(\frac{1}{4} + \frac{1}{2} + \frac{1}{8}\right) \div 3$, which equals $\frac{7}{24}$.

16. There are $1\frac{4}{9}$ times as many white notes as black notes on a full-sized piano keyboard. There are 88 notes altogether. Determine the number of white notes and the number of black notes.



17. Pedro's CDs are stored in three full racks of different sizes. The small rack holds $\frac{1}{2}$ as many CDs as the medium rack. The medium rack holds $\frac{1}{2}$ as many CDs as the large rack. There are 224 CDs altogether. How many are in each rack?

MATH LINK

About $\frac{1}{4}$ of the species of mammals that live in Canada can be found in the Taiga Shield ecozone. About 50 species of mammals can be found in this ecozone. How many species of mammals in Canada live outside the Taiga Shield ecozone?

