

11.1

Determining Probabilities Using Tree Diagrams and Tables

Focus on...

After this lesson, you will be able to...

- determine the sample space of a probability experiment with two independent events
- represent the sample space in the form of a tree diagram or table
- express the probability of an event as a fraction, a decimal, and a percent



At the end of a unit on probability, Ms. Pascal decided to allow her students to determine what kind of test the class would write. All the students' names were put into a hat. Owen was chosen to spin a spinner divided into three equal regions to determine the kind of test: multiple choice (MC), short answer (SA), or a combination (MC & SA). Ava was chosen to roll a four-sided die to determine the number of questions on the test: 5, 10, 15, or 20.

Ms. Pascal explained that spinning the spinner and rolling the die are **independent events**. How does she know that these events are independent?

independent events

- results for which the outcome of one event has no effect on the outcome of another event

Explore the Math

How can you use the outcomes of an experiment to determine probabilities?

1. Show how you could represent the possible outcomes of this experiment.
2. What is the **probability** that the test will have multiple-choice questions only? How did you determine your answer?

Materials

- ruler

probability

- the likelihood or chance of an event occurring

3. What is the probability that the test will consist of ten questions? Explain your reasoning.
4. List the **sample space** for this experiment.

sample space

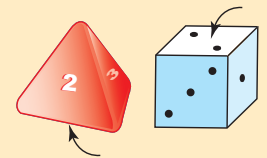
- all possible outcomes of a probability experiment

Reflect on Your Findings

5. Show your answers to parts b), c), and d) as a fraction, a percent, and a decimal.
 - a) How many different tests are possible for the students in Ms. Pascal's class?
 - b) What is the probability that the students will write a combined multiple-choice/short-answer test with 20 questions? Show how you arrived at your answer.
 - c) What is the probability that students will write a multiple-choice test with at least ten questions?
 - d) What is the probability that the students will not write a short-answer test with 15 questions? Explain how you found your answer.

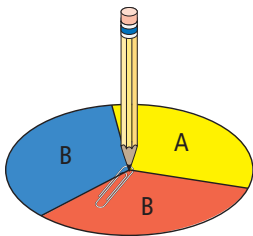
Did You Know?

When you roll a four-sided die, you read the number that is on the bottom. When you roll a six-sided die, you read the number on top.



Example 1: Determine Probabilities From a Tree Diagram

A spinner is divided into three equal regions as shown. The spinner is spun twice. For each probability you determine, express the answer as a fraction, a decimal, and a percent.



- a) What is the probability of spinning A on the first spin?
- b) Draw a tree diagram to represent the sample space for both spins.
- c) What is the probability of spinning A followed by B: $P(A \text{ then } B)$?
- d) What is the probability of getting the same letter on both spins: $P(A, A)$ or $P(B, B)$?

favourable outcome

- a successful result in a probability experiment

Solution

- a) The spinner has three equal regions: A, B, and B. There is only one favourable outcome, A, out of the three regions.

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$$P(A) = \frac{1}{3} \\ = 0.\bar{3}$$

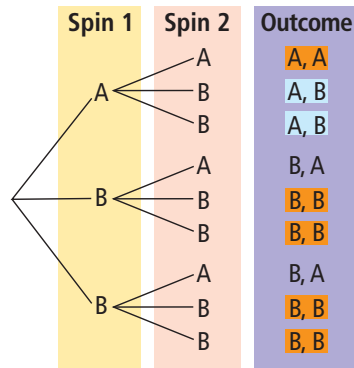


The probability of spinning an A is $\frac{1}{3}$, $0.\bar{3}$, or $33.\bar{3}\%$.

Strategies

Draw a Diagram

- b) The following tree diagram displays all possible outcomes.



- c) The tree diagram shows nine possible outcomes. There are two favourable outcomes (shaded blue).

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$$P(\text{A then B}) = \frac{2}{9} \\ = 0.\bar{2}$$



The probability of spinning A on the first spin and B on the second spin is $\frac{2}{9}$, $0.\bar{2}$, or $22.\bar{2}\%$.

- d) The favourable outcomes (shaded orange) in the tree diagram are (A, A), (B, B), (B, B), (B, B), (B, B). The probability that the same letter will appear on both spins is $\frac{5}{9}$, $0.\bar{5}$, or $55.\bar{5}\%$.

Since $\frac{2}{10}$ is 20%, **M E** the answer should be slightly greater than 20%.

Show You Know

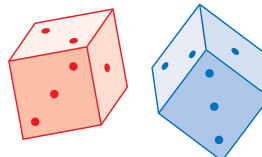
Ellen flips a coin and rolls a four-sided die numbered 1, 2, 3, and 4.



- What is the sample space? Use a tree diagram to show how you got your answer.
- What is $P(H, 4)$?

Example 2: Determine Probabilities From a Table

Two standard six-sided dice are rolled. One die is blue and the other is red. For each probability you determine, express the answer as a fraction, a decimal, and a percent.



- Create a table to represent the sample space.
- What is the probability of rolling a sum greater than ten?
- What is the probability that the number on the red die is one larger than the number on the blue die?
- What is the probability that the sum of the two numbers is less than 11?

Solution

- The following table represents the sample space. The numbers from the red die are shown in red and the numbers from the blue die are shown in blue.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Strategies

Make a Table

- b) The probability of rolling a sum greater than ten can be found by adding the two numbers in each cell of the table. There are three cells in the table with a sum greater than ten. So, there are three favourable outcomes.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

$$P(\text{sum} > 10) = \frac{3}{36}$$

$$= 0.08\bar{3}$$

$$\boxed{3} \div \boxed{36} = 0.08333333$$

The probability of a sum greater than ten is $\frac{3}{36}$, $0.08\bar{3}$, or $8.\bar{3}\%$.

- c) The probability that the number on the red die will be one larger than the number on the blue die can be found by counting favourable outcomes in the table.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

$$P(\text{number on red die is one larger than number on blue die}) = \frac{5}{36}$$

$$= 0.13\bar{8}$$

$$\boxed{5} \div \boxed{36} = 0.13888889$$

The probability that the number on the red die is one larger than the number on the blue die is $\frac{5}{36}$, $0.13\bar{8}$, or $13.\bar{8}\%$.

- d) You can find the probability that the sum of the two numbers will be less than 11 by counting favourable outcomes.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Sometimes it is quicker to count the number of non-favourable outcomes and then subtract this number from the total number of possible outcomes. In this example, a non-favourable outcome is a sum greater than 10. There are three non-favourable outcomes.
 $36 - 3 = 33$
 favourable outcomes.

$$P(\text{sum} < 11) = \frac{33}{36} = 0.91\bar{6}$$

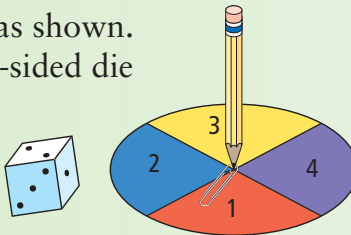
C
33
÷
36
=
0.91666666

The probability that the sum of the two numbers is less than 11 is $\frac{33}{36}$, $0.91\bar{6}$, or $91.\bar{6}\%$.

Show You Know

A spinner is divided into four equal regions as shown. You spin this spinner and roll a standard six-sided die once each.

- Create a table to show the sample space.
- What is $P(4, 4)$?
- What is $P(\text{sum} > 5)$?

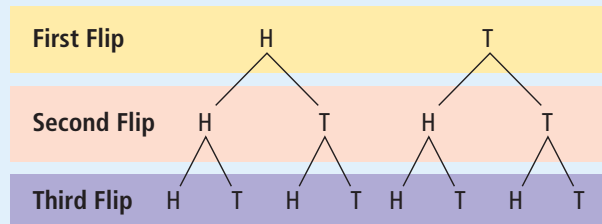


Key Ideas

- Probability = $\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$
- The probability of both A and B occurring can be expressed as $P(A, B)$.
- The probability of event A occurring followed by event B can be expressed as $P(A \text{ then } B)$.
- You can use tree diagrams and tables to show the sample space for a probability experiment.
- Probabilities can be determined from tree diagrams and tables by direct counting of favourable outcomes and comparing the number of favourable outcomes with the total number of outcomes.

Communicate the Ideas

- John flips a coin and rolls a standard six-sided die.
 - What does the notation $P(H, 3)$ mean?
 - Explain how you could use a tree diagram to determine $P(H, 3)$.
- Monique missed class today. Explain to her how you could use this tree diagram to determine the probability of flipping a coin three times and getting exactly two heads and one tail.



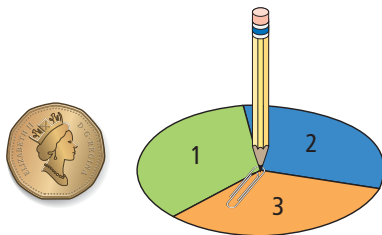
Check Your Understanding

Practise

Express all probabilities as a fraction, a decimal, and a percent.

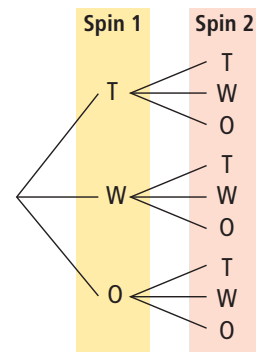
For help with #3 and #4, refer to Example 1 on pages 411–412.

- A spinner is divided into three equal regions as shown. Damien flips a coin and spins the spinner once.



- Draw a tree diagram to represent the sample space.
- List the sample space.
- What is the probability of $P(H, 2)$?

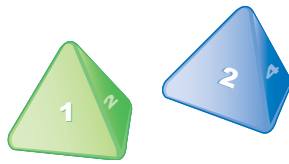
- The following tree diagram represents the sample space for a probability experiment.



- What is the sample space for this experiment?
- What is $P(T, W)$?
- What is the probability that both letters are identical?

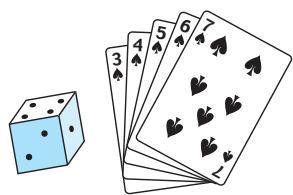
For help with #5 and #6, refer to Example 2 on pages 413–415.

5. Two four-sided dice are each rolled once. Each die is numbered 1, 2, 3, and 4.



- Create a table to represent the sample space.
- What is the probability that the sum is greater than five?
- What is the probability that the same number is the outcome on both dice?

6. Ali draws a card at random from the set of five cards pictured and rolls a standard six-sided die once.



- Create a table to show the sample space.
- What is the probability that the same number is the outcome on both the card and die?
- What is the probability that the sum of the two numbers is even?
- What is the probability that the number on the die is equal to or larger than the number on the card?

Apply

7. Lucy is jigging for fish through the ice. She has an equal chance of catching a whitefish, a trout, an arctic char, or losing the fish. If she pulls her hook out twice, what might she catch?
- Draw a table showing the results of Lucy's fishing.
 - What is $P(\text{whitefish, char})$ in either order?
 - What is $P(\text{char, char})$?
 - What is the probability she will catch nothing at all?

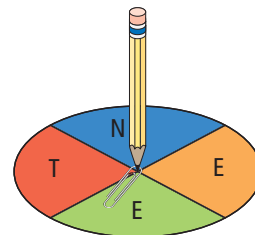
8. The sample space for the flip of a coin and a randomly picked card from five playing cards is (H, 6), (H, 7), (H, 8), (H, 9), (H, 10), (T, 6), (T, 7), (T, 8), (T, 9), and (T, 10).

- Draw a tree diagram to show the sample space.
- Construct a table to show the sample space.
- What is the probability that the result of this experiment includes an even-numbered card?

9. Two babies were born today.

- Construct a table to show the possible genders for the two babies.
- What is the probability that there is one boy and one girl?
- What assumption did you make about the likelihood of a boy or girl being born?

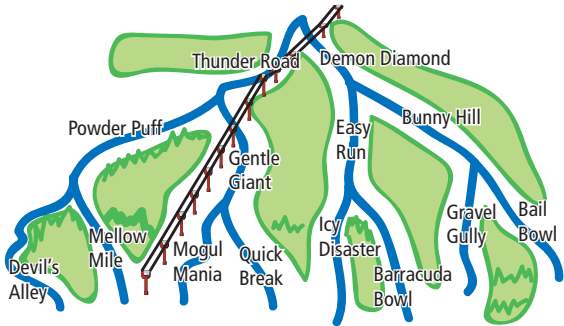
10. A spinner is divided into four equal regions. The spinner is spun twice.



- Create a table to show the sample space.
- What is the probability of spinning a T and then an E: $P(\text{T then E})$?
- What is $P(\text{E, E})$?
- What is $P(\text{same letter on both spins})$?

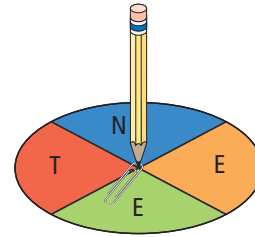
Extend

11. Nick and Manny are snowboarding in the Rockies. On one run down the mountain, they decide to flip a coin to choose which of two paths they will take at each of the three places where the ski runs branch. They will go down the left ski run if the coin is a head and the right ski run if the coin is a tail.



- What is the probability that they will take Thunder Road?
- What is the probability that Nick and Manny will finish on a run containing the name *Bowl*?
- What is the probability that they will take Thunder Road and Quick Break? Explain your answer.

12. A spinner is divided into four equal regions. The spinner is spun three times.



- Draw a tree diagram to show the sample space.
 - What is the probability of $P(E, E, E)$?
 - What is the probability of spinning three different letters in alphabetical order?
 - What is the probability that one letter appears exactly twice?
13. Alena rolls two standard six-sided dice.
- What is the probability that the difference between the two numbers is two?
 - What is the probability that the sum is a multiple of three?
 - What is the probability that the product is a multiple of four?

MATH LINK

The stick game uses four flat sticks. One side of each stick is bare and the other side is decorated. The four sticks are tossed in the air and allowed to fall to the ground. The score depends on the number of decorated sides that land facing up.

- Draw a tree diagram or create a table to show the possible outcomes.
- At the end of each branch or in each cell, record the total number of decorated sides showing.
- What is the probability of exactly three sticks landing decorated side up?



Did You Know?

Originally, rib bones from a buffalo or deer were used for the stick game.